Agricultural Sciences, 7(2):23-28 (2002)

Maintaining Fresh-Water Aquifers Over Saline Water in Coastal Aquifers

E.G. Youngs

Institute of Water and Environment, Cranfield University,
Silsoe, Bedford MK45 4DT England

المحافظة على طبقات المياه العذبة الجوفية فوق الماء المالح في الطبقات الجوفية الساحلية

ادوارد يونجس

خلاصـة: تـم استخدام تحليل النزّ لتحري العلاقة بين موارد ومصادر الإنتاج للمياه العذبة على إبقاء العمق اللازم للمياه فوق طبقات الماء المالح وتجنب تداخل المياه المالحة مع المياه العذبة في مناطق المياه الجوفية الساحلية في الظروف الثابتة . الأمثلة المعطاة في هذه الدراسة توضح بأن الأعماق القصوى يمكن وصولها عندما يكون المصدر بالقرب من الحوض المائي والمورد بجانب الساحل .

ABSTRACT: Seepage analysis is used to investigate the location dependence of fresh-water inputs and outputs in maintaining the depth of the fresh water over the saline water intruding from the sea in coastal aquifers in steady state conditions. Examples are given that show that maximum depths occur when the inputs are in the vicinity of the watershed and the outputs are near the coast.

Keywords: Saline water intrusion, coastal aquifers, steady state, Ghyben-Herzberg lens, groundwater abstraction, seepage.

Fresh-water aquifers overlying saline water near coasts are maintained by water supplied by surface infiltration or by injection into wells. Relatively small rates of fresh water can maintain large bodies of fresh-water and limit the amount of sea-water intrusion. However, exploitation of the freshwater aquifer reduces the depth of freshwater and can result in brackish water supplies from pumped wells. Analyses of the movement of the fresh water under given conditions give the depth of the fresh-water lens at a given position. A complete analysis must consider the non-steady-state situation of the changing position of the interface between fresh-water and saline water regions considering also the movement of the saline water to and from the sea. Here we consider the simpler steady-state situation when the interface remains stationary with no movement of saline water, to give an insight into the extent of the fresh-water region. Examples using steady-state theory show the dependence of the depth of the fresh-water lens above the saline water on the location of inputs and outputs.

Steady–State Analysis of Fresh-Water Aquifer

In the analysis of the so-called Ghyben-Herzberg lens of fresh water overlying saline water intruding from the sea in coastal aquifers (Childs, 1969; Bear, 1972), the assumption is made that no mixing takes place between the saline water and fresh water so that a sharp interface exists between the fresh-water region above and the saline water below in communication with the sea (Fig.1). The upper boundary of the fresh-water aquifer is a water table where the soil-water pressure is atmospheric and the lower is the boundary between fresh and saline water where the pressure in the fresh water is the same as the pressure in the saline water. If Z is the vertical coordinate of the interface measured from sea level, ρ_s the density of the saline water, and g the acceleration due to gravity, the soil-water pressure at the interface is $p_Z = -Z\rho_s g$.

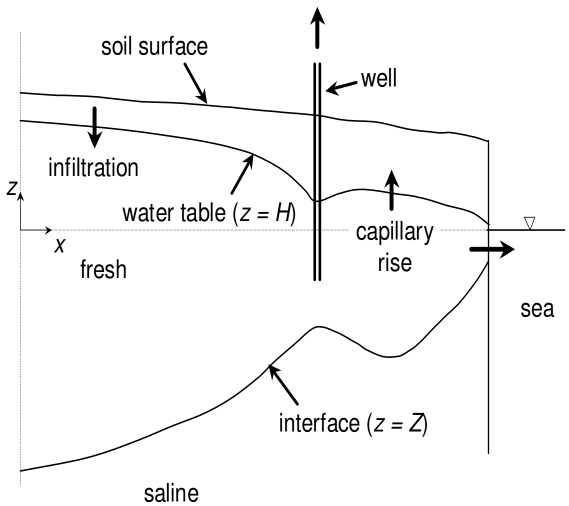

Figure 1. Fresh water overlying saline water in coastal regions (not to scale).

Seepage analysis (Youngs, 1965, 1966, 1986) gives the component of horizontal seepage of fresh water, Q_x, at position (x,y) in the x-direction as (Youngs,1971a):

$$Q_x = -\int_Z^H K \frac{\partial h}{\partial x} dz \tag{1}$$

where K is the hydraulic conductivity of the soil (assumed uniform here although the seepage analysis allows K to vary with depth), $h = p/\rho_f g + z$ the hydraulic head at a height z where the soil-water pressure is p, ρ_f the density of freshwater, and H and Z the water-table height and the level of the interface between fresh and heavier saline water at position (x.y), both measured from sea level. Thus:

$$Q_x = -\int_Z^H \frac{K}{\rho_f g} \frac{\partial p}{\partial x} dz = -\frac{\partial}{\partial x} \int_Z^H \frac{Kp}{\rho_f g} dz - \frac{p_Z K}{\rho_f g} \frac{\partial Z}{\partial x} \tag{2}$$

since p = 0 at z = H at the watertable and $p = p_z$ at z = Z at the interface, $p_z = Z\rho_s g$, so that:

$$Q_x = -\frac{\partial G}{\partial x} \tag{3}$$

where G is the seepage potential defined by:

$$G = \int_Z^H \frac{pK}{\rho_f g} dz - \frac{K\rho_s Z^2}{2\rho_f} \tag{4}$$

Similarly, the horizontal seepage Q_y in the y-direction is:

$$Q_y = -\frac{\partial G}{\partial y} \tag{5}$$

In steady-state conditions, if the vertical downward flux of fresh water through the watertable at (x,y) is q(x,y):

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = q(x, y) \tag{6}$$

so that G is described by Poisson's equation:

$$\frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2} = -q(x, y) \tag{7}$$

At the coast where fresh water seeps into the sea, G = 0 and at a watershed where there is no horizontal seepage, $\nabla G = 0$.

From Equation 4, the value of G is an indication of the thickness of the fresh-water lens. If we assume the Dupuit-Forchheimer approximation of horizontal flow with head constant with depth, we have:

$$G \approx \frac{(\rho_s/\rho_f)}{(\rho_s/\rho_f - 1)} \frac{KH^2}{2} = (\rho_s/\rho_f)(\rho_s/\rho_f - 1)\frac{KZ^2}{2} \tag{8}$$

so that the thickness T of the fresh-water lens is:

$$T = H - Z = \sqrt{\frac{(\rho_s/\rho_f)}{(\rho_s/\rho_f - 1)} \frac{2G}{K}} \tag{9}$$

Fresh-Water Lens Maintained by Uniform Infiltration

With uniform steady infiltration q over a coastal strip of width L and with no water abstraction, all the water supplied over the area by infiltration drains to the sea. G at any distance x from the watershed is then obtained by solving the one-dimensional form of Equation 7 with dG/dx = 0 at x =0 and G = 0 at x = L, giving:

$$G = q(L^2 - x^2)/2 \tag{10}$$

so that G_0, the value of G at the watershed at x = 0, is $qL^2/2$. The seepage analysis (Youngs, 1965, 1966) allows estimates of water-table heights to be made within calculated limits. For this situation of uniform steady infiltration over a coastal strip, Youngs (1971a)

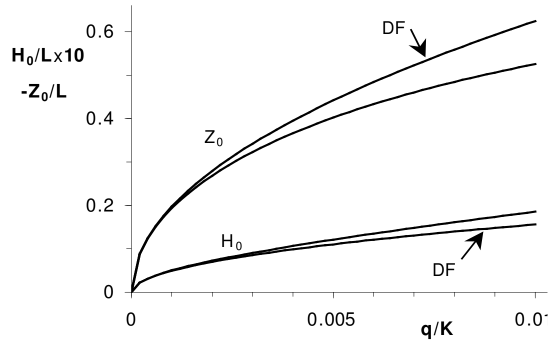

Figure 2. The variation of H_0/L and $-Z_0/L$ with q/K as given by Equations 11 and 12.

argued that streamlines at the watershed diverged so that the hydraulic head gradient decreased with depth, leading to:

$$\sqrt{\frac{(q/K(\rho_s/\rho_f - 1)}{\rho_s/\rho_f}} < \frac{H_0}{L} < \sqrt{\frac{(q/K(\rho_s/\rho_f - 1 + q/K)}{(\rho_s/\rho_f)(1 - q/K)}} \quad (11)$$

and

$$\sqrt{\frac{q/K}{(\rho_s/\rho_f)(\rho_s/\rho_f - 1)}} > \frac{-Z_0}{L} > \sqrt{\frac{q/K(1 - q/K)}{(\rho_s/\rho_f)(\rho_s/\rho_f - 1 + q/K)}} \quad (12)$$

where H_0 and Z_0 are the values of H and Z at x = 0.

As shown in Figure 2, the upper values of H_0 and $-Z_0$ given by Equations 11 and 12 are about 10% above the lower values at q/K = 0.005 and less for smaller values of q/K. The first terms in Equations 11 and 12 are the expressions given by applying the approximate Dupuit-Forchheimer analysis to the problem (Equation 8). Thus, this approximate analysis provides a good estimate of the thickness T of freshwater at the watershed for small steady infiltration rates, as given by Equation 9. Due to the relatively small difference in density between fresh and saline water, a large thickness of fresh water, most of which is below sea level, can be obtained with only a small continuous supply from above. The shape of the fresh-water lens calculated from Equation 10 assuming Dupuit-Forchheimer approximations is shown in Figure 3 in dimensionless variables.

Distributed Input of Fresh Water

The fresh-water aquifer in coastal regions is usually supplied by water non-uniformly over the area, not by uniform infiltration as considered above. The seepage analysis allows a consideration of the effect of location of the freshwater supply on the fresh-water region.

We consider infiltration supplied at a rate q through a surface strip parallel to the coast between L' and L" measured from the watershed with no abstraction of fresh water from the fresh-water aquifer. Solving Equation 7, we obtain the value of G, G_0, at the watershed at x = 0, as:

$$G_0 = q(L'' - L')[L - (L' + L'')/2] = Q_s[L - (L' + L'')/2] \quad (13)$$

where Q_s is the seepage of fresh water per unit width draining to the sea, which is equal to the total amount of water being supplied per unit width over the strip. Equation 13 shows that G_0 is greatest when the infiltration is concentrated in the vicinity of the watershed and is least when concentrated near the sea. Thus, from Equation 9, it follows that the thickness of the fresh-water lens is also greatest when the infiltration is supplied in the vicinity of the watershed.

The fresh-water aquifer can also be maintained by water supplied by injection into wells. For a series of m wells in a coastal aquifer with the nth well located at a distance L_n from the watershed being injected at a rate Q_n, from seepage analysis and the mean value theorem of potential theory (Youngs, 1970) $(G_{av})_0$, the mean value of *G* along the watershed, is given by:

$$(G_{av})_0 = 1/B\left[\sum_{n=1}^{m}(L - L_n)Q_n\right] \quad (14)$$

where B is the length of the coastal strip. As with the case of infiltration through the soil surface, Equation 14 indicates that maximum thickness of the fresh-water lens is obtained when the injection wells are located near to the watershed. Youngs (1971b) applied

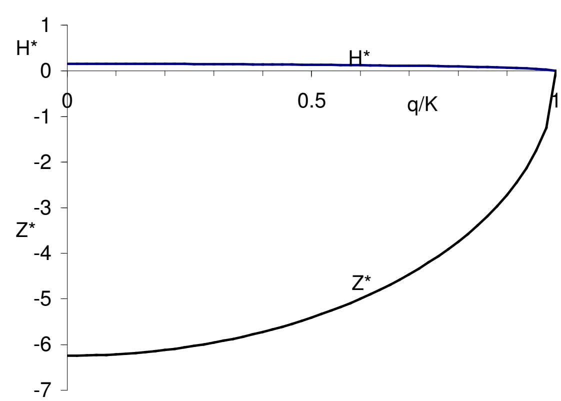

Figure 3. The Dupuit-Forchheimer estimate of the fresh-water lens with uniform steady infiltration, shown as $H^* = (H/L)\sqrt{(K/q)}$ and $Z^* = (Z/L)\sqrt{(K/q)}$ as functions of x/L.

methods used in potential theory to investigate optimum pumping conditions to avoid the upconing of saline water below the well providing an output of brackish water. The analysis allows values of Gat any position (x,y) to be obtained and shows that for maximum supply of fresh water wells should be sunk to sea level and pumped with the level of water in them negligibly small.

Abstraction of Fresh Water

Water abstraction from the fresh-water lens can occur either from pumped wells or by capillary rise to supply surface evaporation. A fresh-water lens can be maintained with given fresh-water inputs and abstractions so long as $dG/dx < 0$ at $x = L$. If the loss is sufficiently large, then $dG/dx > 0$ at $x = L$ and sea-water intrusion will result.

The rate of fresh-water seepage towards the sea between wells at distances L_p and L_{p+1} from the watershed in a coastal strip is described generally by:

$$\frac{dG_{av}}{dx} = -\int_0^x q(x')dx' - 1/B\sum_{n=1}^{p} Q_n, \quad L_p < x < L_{p+1} \tag{15}$$

where G_{av} is the mean value of G at distance x from the watershed. The surface flux distribution, q(x), is negative when there is capillary rise and Q_n is positive for injection and negative for abstraction. Since G = 0 along x = L, integration of Equation 15 yields:

$$\begin{aligned} G_{av} &= \int_x^L\int_0^x q(x')dx'dx + 1/B\int_x^L \sum_{n=1}^{p} Q_n dx \\ &= \int_x^L\int_0^x q(x')dx'dx + \\ &\quad + 1/B\begin{bmatrix} (L_{p+1}-x)\sum_{n=1}^{p} Q_n + (L_{p+2}-L_{p+1})\sum_{n=1}^{p+1} Q_n + \\ (L_{p+3}-L_{p+2})\sum_{n=1}^{p+2} Q_n + \ldots + (L-L_m)\sum_{n=1}^{m} Q_n \end{bmatrix} \\ &= \int_x^L\int_0^x q(x')dx'dx + 1/B\begin{bmatrix} -x\sum_{n=1}^{p} Q_n - L_{p+1}Q_{p+1} - L_{p+2}Q_{p+2} \\ -L_{p+3}Q_{p+3} - \ldots - L_m Q_m + L\sum_{n=1}^{m} Q_n \end{bmatrix} \end{aligned}$$

which reduces:

$$G_{av} = \int_x^L\int_0^x q(x')dx'dx + 1/B\left[\sum_{n=1}^{m}(L-L_n)Q_n - \sum_{n=1}^{p}(x-L_n)Q_n\right] \tag{16}$$

Equation 16 gives the mean value of G at a distance *x* from the watershed for a surface infiltration distribution q(x) and for injection and abstraction from a system of wells. It is seen that the result is dependent on the distribution of the sources and sinks. In the following examples we employ the analysis given here to a variety of situations where water is supplied by infiltration and injection wells and abstracted by capillary rise with loss of water through evaporation and pumped wells.

EXAMPLE 1: $q(x) = q,\ 0 < x\ L';\ q(x) = -e,\ L' < x < L.$

This is the situation where there is uniform infiltration over a strip adjacent to the watershed and capillary rise with loss of water through evaporation over the remaining area. Equation 16 gives the uniform value of G along x = 0 as:

$$G_0 = L'(L - L'/2)q - (L - L')^2 e/2 \tag{17}$$

Since it can be argued that the streamlines near *x* = 0 diverge with depth, Equations 11 and 12 apply and the thickness of the fresh-water aquifer here is known within limits. For small values of q/K it is given to a good approximation by Equation 9. If all the water supplied by infiltration is equal to that abstracted by the evaporation so that qL′ = e (L-L′), , no water seeps into the sea and dG/dx = 0 along x = 0; then:

$$G_0 = LL'q/2 = L(L - L')e/2 \tag{18}$$

The variation of G with the distance *x* from the watershed, both shown in dimensionless units, for various values of e/q when L′ = 0.5, is shown in Figure 4.

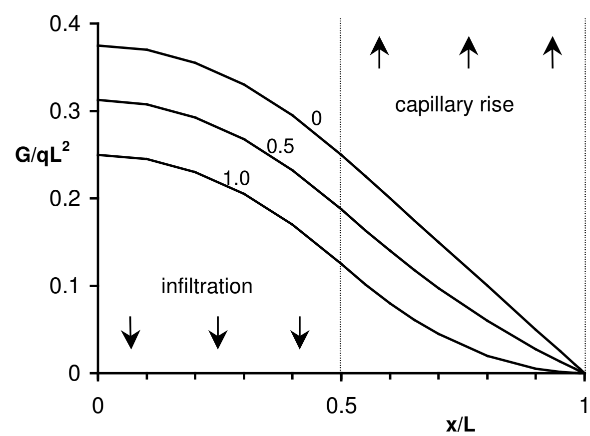

Figure 4. The variation of G/qL^2 with x/L or L’ =0.5 for three values of e/q shown by the curves.

EXAMPLE 2: Line of injection wells at $x = L_i$, total supply rate Q_i over length B of coastal strip, and a line of abstraction wells at $x = L_a$, total abstraction rate Q_a.

For a line of wells at distance L_i from the watershed injecting water at a rate Q_i into the freshwater aquifer and another line of wells at distance L_a abstracting water at a rate Q_a, Figure 5 illustrates the variation of G_{av} with distance x given by Equation 16.

The value of G_{av} at the watershed, $(G_{av})_0$, is:

$$(G_{av})_0 = [(L - L_i)Q_i - (L - L_a)Q_a]/B \quad (19)$$

Equation 19 shows the advantage of supplying water near the watershed and abstracting water from the fresh-water region as near the coast as possible to obtain maximum depth of fresh water.

EXAMPLE 3: Infiltration at a rate q, $L' < x < L''$, line of abstraction wells at $x = L_a$, total abstraction rate Q_a over length B of coastal strip.

Figure 6 illustrates the maintenance of the fresh-water lens by infiltration over a region when water is abstracted by wells, as obtained from Equation 16. Again calculations of Equation 16 show the importance of the location of the sources and sinks with maximum depth of fresh water occurring when the water is supplied in the vicinity of the watershed and abstracted near the sea, with values of $(G_{av})_0$ given by:

$$(G_{av})_0 = (L'' - L')[L - (L'' + L')/2]q - (L - L_a)Q_a/B \quad (20)$$

EXAMPLE 4: Line of injection wells at $x = L_i$, total supply rate Q_i over length B of coastal strip, evaporation from capillary rise at rate $q = e$, $L' < x\ L''$.

Results calculated from Equation 16 for this situation are illustrated in Figure 7. Maximum depth of fresh water results when water supply is in the vicinity of the watershed and abstraction is near the sea, as in previous examples. In this case $(G_{av})_0$ is given by:

$$(G_{av})_0 = (L - L_i)Q_i/B - (L'' - L'\}[L - (L'' + L')/2]e \quad (21)$$

Discussion

Theoretical analyses show that the Ghyben-Herzberg lens of fresh water in coastal aquifers can extend to great depths when maintained by a relatively small input of fresh water. However, exploitation of the fresh water aquifer for domestic, agricultural and industrial uses causes the saline water below to cone upwards into the fresh-water region at the positions of abstraction, and capillary rise with loss of water by evaporation considerably reduces the depth of fresh water. Fresh-water inputs and abstractions are generally intermittent and hence non-steady state theory needs to be used to obtain an accurate estimate of the changing

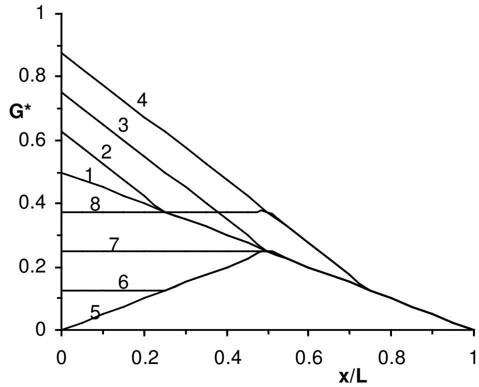

Figure 5. The variation of $G^* = G_{av}B/Q_iL$ with x/L when there are injection wells at $x = L_i$ and abstraction wells at $x = L_a$ for $Q_a/Q_i = 0.5$. Curves 1, 2, 3 and 4, $L_i/L = 0$; curves 5, 6, 7 and 8, $L_i/L = 0.5$; curves 1 and 5, $L_a/L = 0$; curves 2 and 6, $L_a/L = 0.25$; curves 3 and 7, $L_a/L = 0.5$; and curves 4 and 8, $L_a/L = 0.75$.

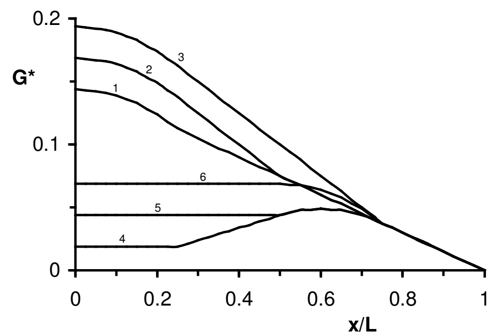

Figure 6. The variation of $G^* = G_{av}/qL^2$ with x/L when there is infiltration at a rate q on to a surface strip $L' < x\ L''$ and abstraction from wells at $x = L_a$ at a rate Q_a. Curves 1, 2 and 3, $L'/L = 0$, $L''/L = 0.5$; curves 4, 5 and 6, $L'/L = 0.5$, $L''/L = 1.0$; curves 1 and 4, $L_a = 0.25$; curves 2 and 5, $L_a = 0.5$; and curves 3 and 6, $L_a = 0.75$:$Q_a/BL_q = 0.1$.

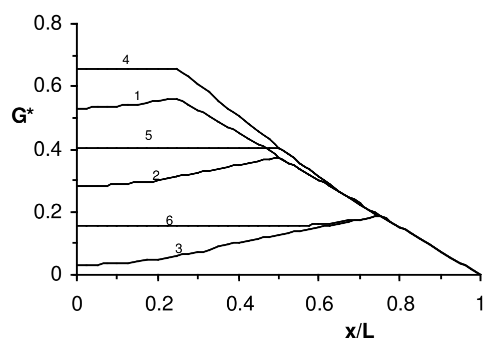

Figure 7. The variation of $G^* = G_{av}/eL^2$ with x/L when water is injected into wells at $x = L_{at}$ a rate Q_i and there is capillary rise with loss of water due to evaporation at a rate e between $L' < x < L''$. Curves 1, 2 and 3, $L'/L = 0$, $L''/L = 0.5$; curves 4, 5 and 6, $L'/L = 0.5$, $L''/L = 1.0$; curves 1 and 4, $L_i = 0.25$; curves 2 and 5, $_{Li} = 0.5$; and curves 3 and 6, $_{Li} = 0.75$:$Q_i/BL_e = 1.0$.

depth of the fresh-water region, taking into account the movement of saline water to and from the sea as well as the movement of fresh water. However, if the intermittent pattern of water input and abstraction is regular with time, average rates used in simpler steady-state analyses should give an estimate of the average extent of the fresh-water region.

Seepage analysis (Youngs, 1965, 1966) has been used to investigate the steady horizontal seepage of fresh water in the Ghyben-Herzberg lens (Youngs, 1971a; 1971b). This shows that the Dupuit-Forchheimer analysis provides a good estimate of the depth of fresh water overlying saline water intruding from the sea in coastal regions for small steady-state uniform infiltration rates over the surface area. In situations where the surface input is not uniform over the whole coastal area, the distribution of the seepage potential defined by Equation 4 can be calculated over the coastal area for the given distribution of inputs and outputs. If this is assumed to relate to the Dupuit-Forchheimer estimate of the depth of fresh water through Equation 9, it provides a means of comparing different distribution patterns of inputs and outputs for maintaining fresh-water coastal aquifers. The various examples given in this paper show how the seepage potential varies in a coastal strip with different distributions of infiltration and capillary rise as well as for different distributions of input and abstraction wells. Maximum values of the seepage potential occur when water input is in the vicinity of the watershed and abstraction takes place near the coast Thus it can be inferred that this pattern of input and abstraction would maximise the depth of the fresh-water lens for given input and abstraction rates.

References

Bear, J. 1972. *Dynamics of Fluids in Porous Media*. New York, Elsevier.

Childs, E.C. 1969. *The Physical Basis of Soil Water Phenomena*. London, John Wiley.

Youngs, E.G. 1965. Horizontal seepage through unconfined aquifers with hydraulic conductivity varying with depth. *J. Hydrology* 3: 283-296.

Youngs, E.G. 1966. Exact analysis of certain problems of groundwater flow with free surface conditions. *J. Hydrology* 4: 277-281

Youngs, E.G. 1970. A mean value theorem in potential theory. *Water Resources Research* 6: 1792.

Youngs, E.G. 1971a. Seepage through unconfined aquifers with lower boundaries of any shape. *Water Resources Research* 7: 624-631.

Youngs, E.G. 1971b. Optimum pumping conditions for wells located in unconfined coastal aquifers. *J. Hydrology* 13: 63-69.

Youngs, E.G. 1986. The analysis of groundwater flows in unconfined aquifers with nonuniform hydraulic conductivity. *Transport in Porous Media* 1:399-417.

Received August 2002.
Accepted November 2002.