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Synthesis of a uniformly Spaced Linear Antenna Array with Symmetric Amplitude Excitation by a Least-Squares Method

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تركيب صف هوائيات خطي بمسافات متساوية بقيمة موجة محفزة متماثلة بواسطة طريقة أقل المربعات الموزنة

جوزيف أ. جيرفس ، وعصام زعبلاوي

خلاصة هذه الورقة تقترح طريقة سهلة غير مكررة لتركيب صف هوائيات خطي بمسافات متساوية بقيمة موجة محفزة متماثلة. النهج المقترح يعتمد على حساب المتجه الفريد لصفوفة موجبة محددة ، ومتماثلة لقيم حقيقية مناسبة. هذه الطريقة يمكن استعمالها لتحديد عوامل موجة التحفيز لصف هوائيات بوجود وصف كامل لنموذج الاشعاع المرغوب وموجات التحفيز المشتقة ستنتج نموذج الاشعاع للهوائي المطلوب بالتحديد أو تقريبا اعتمادا على مدى تعقيد نموذج الاشعاع وعدد عناصر الهوائي. الورقة تعرض عدة امثلة توضح مدى الجدوى العملية وقدرة طريقة

ABSTRACT: A new, non-iterative technique for synthesizing equispaced linear arrays with symmetric excitation is proposed. The method is based on the computation of an eigenvector of an appropriate real, symmetric and positive-definite matrix. This technique can be used to determine, given a complete description of the desired radiation pattern, the array excitation coefficients. These derived excitations will produce, either exactly or approximately, the wanted antenna radiation pattern depending on its complexity and the number of array elements. A number of design examples are presented to illustrate the practicality and the potential of the proposed method.

The objective of antenna array synthesis is to find the antenna configuration in addition to the excitation distribution to achieve desired characteristics in the antenna radiation pattern; such as a prescribed beam pattern response, nulls in specified directions, *etc.* The problem of synthesizing a linear array to achieve a desired radiation pattern (*beam shaping*) has received extensive treatment in the literature (Lo and Lee, 1988 and Rudge *et. al.*, 1986). The classical techniques include the Fourier method (Balanis, 1982) in which a given radiation pattern is approximated by a Fourier partial sum and the Woodward sampling method (Balanis, 1982, and Lee and Mostafavi, 1996) in which the synthesized radiation pattern is achieved by sampling the desired pattern at specified points. Numerous other synthesis techniques exist such as the iterative sampling method (Stutzman, 1971) or the iterative optimization method (Shpak and Antoniou, 1992). This paper presents a new approach to linear array shaped synthesis. The underlying technique follows closely that used in the design of linear-phase

$$E = \int_R [D(\theta) - A_f(\theta)]^2 \, d\theta \tag{1}$$

where $D(\theta)$ is the desired radiation pattern, $A_f(\theta)$ is the synthesized array factor, θ is the angle measured from the linear array axis (see Fig. (1)), and R is the visible region of the antenna for which $0 \le \theta \le \pi$ (or $-\pi/2 \le \theta \le \pi/2$).

By minimizing E, the excitation coefficients of the $A_f(\theta)$ can be determined. The main disadvantage of the formulation in Eq.(1) is that the resulting error is small in an average sense and not at particular locations on the antenna radiation pattern. To circumvent this, a weighting function can be introduced in the integrand of Eq.(1). Thus

$$E = \int_R \alpha(\theta) \, [D(\theta) - A_f(\theta)]^2 \, d\theta \tag{2}$$

where $\alpha(\theta)$ is some positive weighting function.
E given by Eqs.(1) and (2) can be reformulated in quadratic form as follows

$$E = \mathbf{a}^t \mathbf{P} \mathbf{a} \tag{3}$$

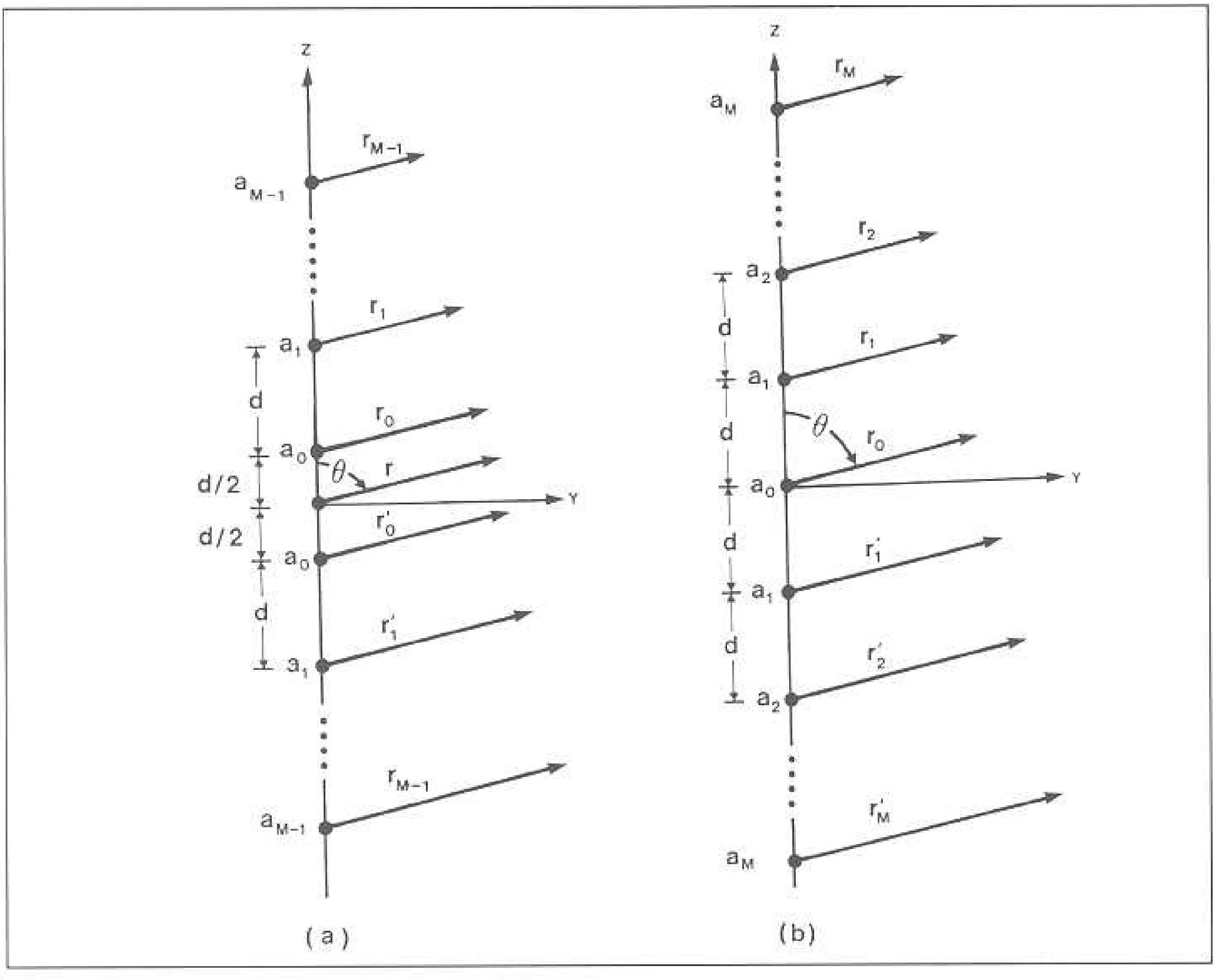


Figure 1. Geometry of the N-element linear array positioned symmetrically along the z-axis; (a) even number of elements, N=2M (b) odd number of elements, N = 2M+1.

where **a** is a real vector representing the synthesized excitation coefficients, $\mathbf{a}^t$ is the transpose of **a** and **P** is a real symmetric and positive-definite matrix whose elements are dependent on the desired radiation pattern. In accordance with Rayleigh principle (Noble and Daniel, 1988), the eigenvector corresponding to the least eigenvalue of **P** minimizes E. The elements of the corresponding eigenvector yield the wanted excitation coefficients.

The synthesis method described here differs from earlier approaches. The Fourier and Woodward beam shaping techniques are inflexible in that if the desired pattern is not achieved, nothing can be done other than to increase the number of array elements. An iterative procedure is followed in Stutzman (1971), which is not the case here. In Sphak and Antoniou (1992), the formulation of the method of synthesis varies as to whether side lobe levels, the beamwidth or nulls are specified. The eigen approach adopted here is general and can incorporate any of the above. In the proposed method, the weighting function can be chosen such as to emphasize the parts of the radiation pattern that have to be synthesized to a high degree of accuracy.

In this paper the simplicity and versatility of the proposed eigen approach for the design of uniformly spaced linear array antennas with symmetric excitation shall be demonstrated. In Section II we formulate the new approach. The design algorithm is given in Section III. Numerical design examples are incorporated in Section IV. Finally, concluding remarks are embodied in Section V.

Problem Formulation

Consider a linear array of N equally spaced elements as illustrated in Fig. 1 with non-uniform excitations. Assuming a symmetrical amplitude excitation about the centre of the array, the array factor for even and odd number of elements can be expressed as

$$A_f(\theta) = \begin{cases} \sum_{n=0}^{M-1} a_n \cos\left[(2n+1)\frac{\pi d}{\lambda}\cos\theta\right] & N(\text{even}) = 2M \quad (4a) \\ a_0 + 2\sum_{n=1}^{M} a_n \cos\left[2n\frac{\pi d}{\lambda}\cos\theta\right] & N(\text{odd}) = 2M+1 \quad (4b) \end{cases}$$

where θ is the angle measured from the array axis, λ is the wavelength, d is the element spacing, and a_n is the excitation of the nth element on either side of the array midpoint.

If the desired array factor is $D(\theta)$, the least-squares method requires formulating an error function E given by Eq.(1) or (2).

The parameters a_n appearing in Eq.(4) are determined by minimizing E. The approach adopted is to formulate E in such a way that the coefficients a_n can be derived from an eigenvector of a real, symmetric and positive-definite matrix. This is done by reformulating Eq.(2) in a quadratic form as given by Eq.(3). The eigenvector corresponding to the smallest eigenvalue of this matrix minimizes E in accordance with Rayleigh principle (Noble and Daniel, 1988). The elements of this eigenvector give the excitation coefficients.

Define

$$\mathbf{a} = [a_0\ a_1\ \dots\ a_{M-2}\ a_{M-1}]^t, \quad (N \text{ even}) \tag{5a}$$

or

$$\mathbf{a} = [a_0\ a_1\ \dots\ a_{M-1}\ a_M]^t, \quad (N \text{ odd}) \tag{5b}$$

and

$$\mathbf{c}(\theta) = [\cos(u)\ \ \cos(3u)\ \dots\ \cos(\{2M-1\}u]^t, \quad (N \text{ even}) \tag{6a}$$

or

$$\mathbf{c}(\theta) = [1\ \ 2\cos(2u)\ \dots\ 2\cos(\{2M\}u)]^t, \quad (N \text{ odd}) \tag{6b}$$

where $$u = (\pi d/\lambda)\cos\theta \tag{6c}$$

The expression for the array factor can be written in the matrix form as

$$A_f(\theta) = \mathbf{a}^t\,\mathbf{c}(\theta) = \mathbf{c}^t(\theta)\mathbf{a} \tag{7}$$

The error can now be written as

$$E = \mathbf{a}^t\left\{\int_0^{\pi}\left[\frac{D(\theta)}{D(\theta_o)}\mathbf{c}(\theta_o) - \mathbf{c}(\theta)\right]\left[\frac{D(\theta)}{D(\theta_o)}\mathbf{c}(\theta_o) - \mathbf{c}(\theta)\right]^t d\theta\right\}\mathbf{a} \tag{8}$$

As is evident in Eq.(8), a normalized factor $c(\theta_o)/D(\theta_o)$ for which $D(\theta_o) \neq 0$ has been added to the desired radiation pattern such that the actual value of the pattern at the reference angle θ_o in the radiation pattern is exactly equal to the desired value. Doing this facilitates the writing of E as a quadratic form in **a** and will lead to the eigen formulation $E = \mathbf{a}^t\mathbf{P}\mathbf{a}$. In this paper, θ_o is chosen to equal $\pi/2$. This is imposed by the symmetry of the excitation. Thus the radiation patterns that have to be synthesized by necessity have to be symmetric about $\pi/2$.

The elements of the matrix **P** are given by

$$p(n,m) = \begin{cases} \int_0^{\pi}\left[\frac{D(\theta)}{D(\theta_o)}\cos\{(2n+1)u_o\} - \cos\{(2n+1)u\}\right]\left[\frac{D(\theta)}{D(\theta_o)}\cos\{(2m+1)u_o\} - \cos\{(2m+1)u\}\right]^t d\theta \quad (9a) \\ \qquad\qquad \text{For } N \text{ even} \\ \int_0^{\pi}\left[\frac{D(\theta)}{D(\theta_o)}\cos\{(2n)u_o\} - \cos\{(2n)u\}\right]\left[\frac{D(\theta)}{D(\theta_o)}\cos\{2(m)u_o\} - \cos\{2(m)u\}\right]^t d\theta \quad (9b) \\ \qquad\qquad \text{For } N \text{ odd} \\ \text{where} \quad u_o = \frac{\pi d}{\lambda}\cos(\theta_o) \quad \text{and} \quad u = \frac{\pi d}{\lambda}\cos(\theta) \end{cases}$$

In Eq.(9) $0 \le n,m \le M-1$ for N even and $0 \le n,m \le M$ for N odd. Thus **P** is a real, symmetric, positive-definite matrix whose order is MxM for N even and (M+1)x(M+1) for N odd. The integrals in the right side of Eq.(9), in general, cannot be expressed in closed form. Thusa numerical method is usually required.

The above formulation assumes that $D(\theta)$ is described by a single function in the region $0 \le \theta \le \pi$. To generalize it, let us assume that $0 \le \theta \le \pi$ is divided into k subregions such that:

$$D(\theta) = \begin{cases} D_1(\theta) & 0 \le \theta \le \theta_1 \\ D_2(\theta) & \theta_1 \le \theta \le \theta_2 \\ \quad . & \\ \quad . & \\ \quad . & \\ D_k(\theta) & \theta_{k-1} \le \theta \le \pi \end{cases} \qquad (10)$$

The total error in this case is given by

$$E_{tot} = \alpha_1 E_1 + \alpha_2 E_2 + ... + \alpha_k E_k \qquad (11)$$

where

$$E_i = \mathbf{a}^t \left\{ \int_{\theta_i}^{\theta_{i+1}} \left[\frac{D(\theta)}{D(\theta_o)} \mathbf{c}(\theta_o) - \mathbf{c}(\theta) \right] \left[\frac{D(\theta)}{D(\theta_o)} \mathbf{c}(\theta_o) - \mathbf{c}(\theta) \right]^t d\theta \right\} \mathbf{a} \qquad (12)$$

and the quantities α_i represent the weighting factors that control the relative accuracy in the different subregions. This is a useful formulation when the desired pattern cannot be exactly synthesized. By varying the weighting factors, the pattern in the region of interest can be achieved. The numerical examples in the subsequent section illustrate this principle. It must be pointed out that the functions must be symmetrical about $\pi/2$ since we are considering symmetric amplitude excitation about $\theta_o = \pi/2$. (If R is chosen to be represented by $-\pi/2 \le \theta \le \pi/2$, then $\theta_o = 0$ and $D(\theta)$ has to be symmetric about $\theta_o = 0$.)

Design Algorithm

For synthesizing linear array antennas to meet a prescribed radiation pattern requirement using the formulation developed in the preceding section, the following steps can be followed to determine the optimal excitation coefficients in the minimal least squares sense.

Step 1: Specify the desired radiation pattern $D(\theta)$ and the corresponding weighting factors α_i.

Step 2: Set a value for N, the number of elements in the array.

Step 3: Compute the elements of matrix **P** using Eq.(9).

Step 4: Determine the smallest eigenvalue of the matrix **P**.

Step 5: Deduce the eigenvector corresponding to this eigenvalue. The elements of this vector represent the excitation coefficients a_n that appear in Eq.(4).

Step 6: Plot. $D(\theta)$ and $A_i(\theta)$ by substituting for the values of a_n obtained in Step 5 in Eq.(4).

Step 7: Decide whether the design criteria have been met. If not, continue as follow.

Step 8: Increase N or modify the weighting factors.

Step 9: Repeat Steps 1 to 8 until the specifications have been achieved within an acceptable tolerance.

Design Examples

A number of examples will be considered in this section to demonstrate the practicality and the potential of the proposed method.

A. SYNTHESIS OF A SECTOR PATTERN: The desired pattern is defined by

$$D(\theta) = \begin{cases} 1, & \text{for} \quad 45^\circ \le \theta \le 135^\circ \\ 0, & \text{otherwise} \end{cases}$$

The desired pattern is synthesized using a 12-element array spaced half a wavelength apart using different weighting factors. The outcome of this synthesis process is displayed in Fig. 2. The ripples and the overshoots in the region of interest ($45^\circ \le \theta \le 135^\circ$) are obvious when equal weighting factors are used i.e. $\alpha_1 = \alpha_2$ represented by the solid curve (α_1 is the weighting factor of the flat topped part and α_2 is that of the zero part). To control the amplitude of the ripples to any specified level, the weighting factors can be chosen to achieve that. Fig. (2) shows two cases where $\alpha_1 = 0.1\alpha_2$ (dotted curve) and $\alpha_1 = 0.01\alpha_2$ (dashed curve). However, as is apparent from the figure, these reductions in the ripples amplitude are achieved at the expense of increasing the beamwidth and side lobe level. The excitation coefficients for the three cases considered are shown in Table 1.

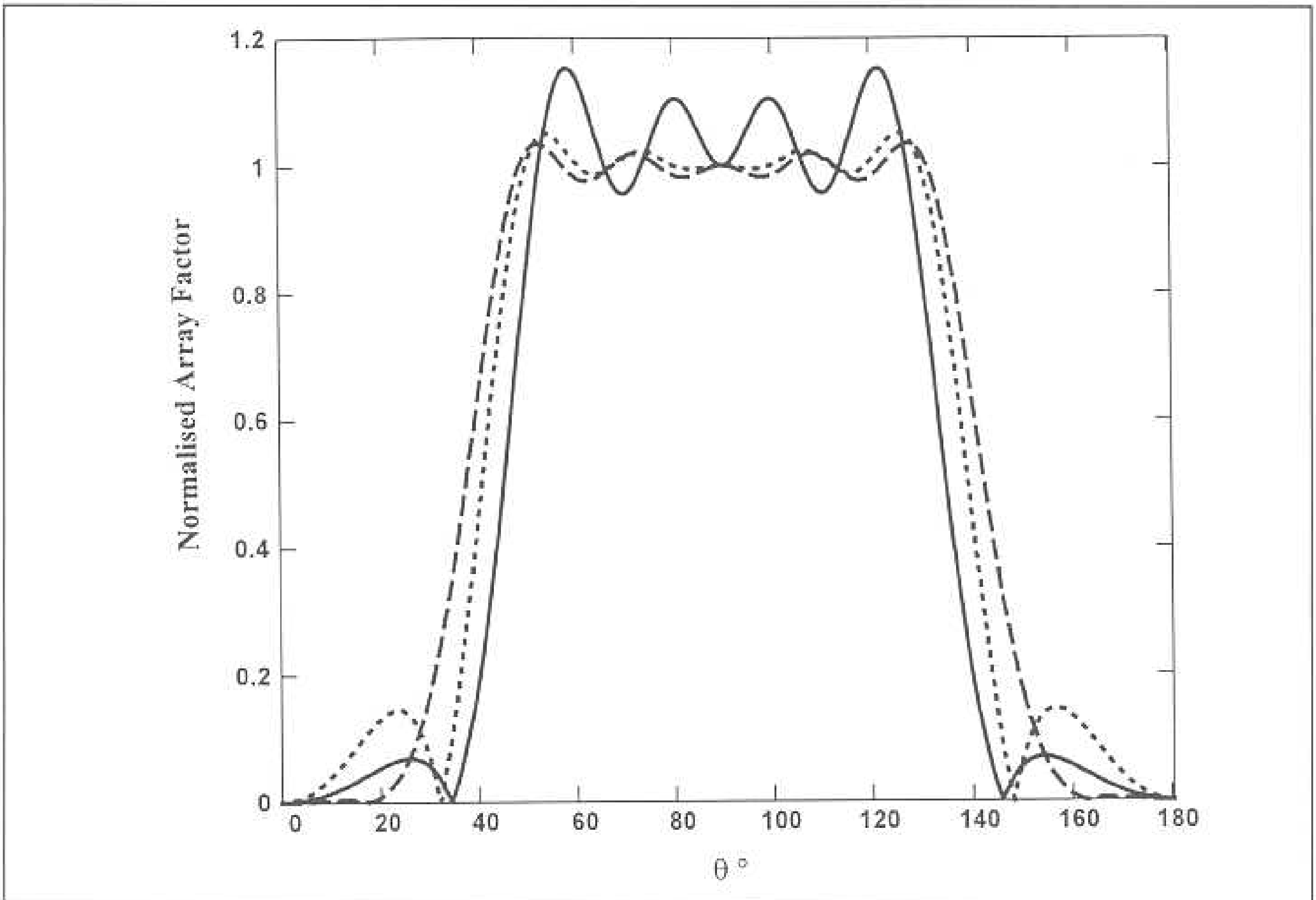


Figure 2. Sector pattern for a 12-element array with half a wavelength inter element spacings obtained using different weighting factors (α_1 for the flat-topped part and α_2 for the zero part). The solid curve represents the pattern obtained with equal weights i.e. $\alpha_1 = \alpha_2$, the dotted curve with $\alpha_1 = 0.1\alpha_2$ and the dashed curve with $\alpha_1 = 0.01\alpha_2$.

TABLE 1

Excitations for a sector antenna.

Weighting Factors		a_n					
α_1	α_2	a_0	a_1	a_2	a_3	a_4	a_5
1	1	1	-0.065	-0.142	0.157	-0.055	-0.052
0.1	1	1	-0.138	-0.085	0.136	-0.110	0.044
0.01	1	1	-0.195	-0.009	0.072	-0.076	0.042

B. ARRAY WITH SPECIFIED MAIN LOBE WIDTH: The main lobe parameters that are specified are the half power beamwidth (HPBW) and the first null beamwidth (FNBW).Since this means that only four points through which the main lobe has to pass are known, there are infinite number of functions that can achieve this. For our case we have defined $D(\theta)$ as follows:

and α_i are the weighting factors (θ is in radians in equations for $D_2(\theta)$ and $D_3(\theta)$).

A 12-element array was chosen to synthesize the above pattern with elements spaced $\lambda/2$ apart for different weighting factors. Fig 3a is a plot of the case where equal weighting factors were used ($\alpha_1=\alpha_2=\alpha_3$). The solid curve shows the synthesized pattern and the dashed curve the desired pattern. With equal weights, the main lobe

$$D(\theta) = \begin{cases} \alpha_1 D_1(\theta) & 0 \le \theta \le \dfrac{\pi}{2} - \dfrac{FNBW}{2} \\ \alpha_2 D_2(\theta) & \dfrac{\pi}{2} - \dfrac{FNBW}{2} \le \theta \le \dfrac{\pi}{2} - \dfrac{HPBW}{2} \\ \alpha_3 D_3(\theta) & \dfrac{\pi}{2} - \dfrac{HPBW}{2} \le \theta \le \dfrac{\pi}{2} + \dfrac{HPBW}{2} \\ \alpha_2 D_2(\theta) & \dfrac{\pi}{2} + \dfrac{HPBW}{2} \le \theta \le \dfrac{\pi}{2} + \dfrac{FNBW}{2} \\ \alpha_1 D_1(\theta) & \dfrac{\pi}{2} + \dfrac{FNBW}{2} \le \theta \le 2\pi \end{cases}$$

where

$$D_1(\theta) = 0$$

$$D_2(\theta) = \frac{\sqrt{2}}{FNBW \quad - HPBW}\left[\frac{FNBW}{2} - \left|\theta - \frac{\pi}{2}\right|\right]$$

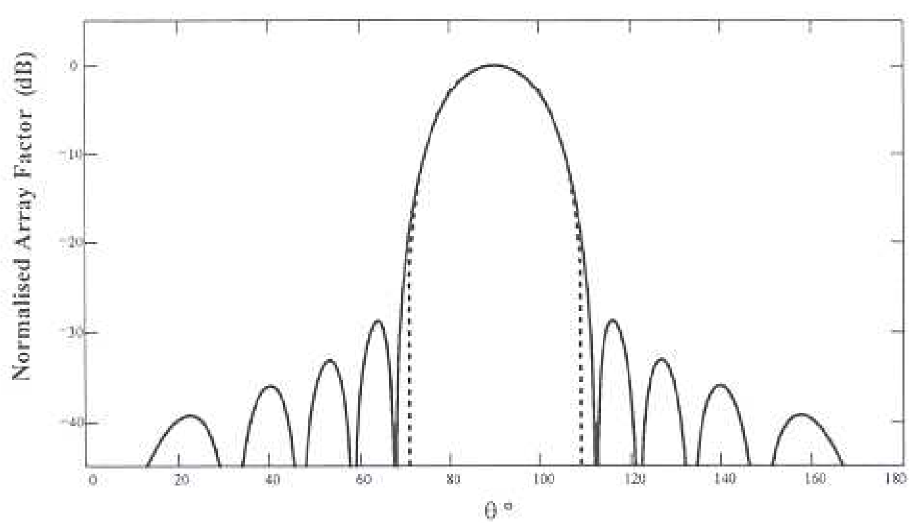
$$D_3(\theta) = \cos\left[\frac{\pi}{2}\left(\frac{\theta - \frac{\pi}{2}}{HPBW}\right)\right]$$

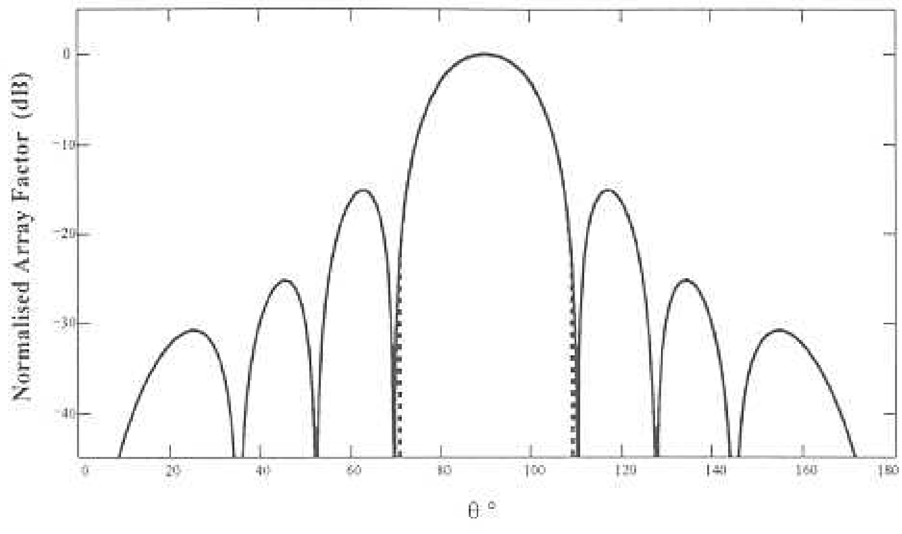
deviates from the desired pattern. To minimize this deviation, the weighting factors were chosen such $\alpha_1 = 0.001\ \alpha_2$ and $\alpha_2 = \alpha_3$. The resulting plot is given in Fig 3b. A comparison between the two plots reveals that the side lobe level has increased from -28dB to -16dB. This means that a compromise has to be struck between synthesizing the main lobe precisely and the acceptable side lobe level. The excitation coefficients obtained are shown in Table 2. To lower this sidelobe even further, the antenna can be synthesised using more elements. The synthesised pattern of a 36-element array is shown in Fig 3c for $\alpha_1 = 0.001\ \alpha_2$ and $\alpha_2 = \alpha_3$. The excitations coefficients are given in Table 3. As noted from this figure, the sidelobe level is now down to -23dB.

TABLE 2

Excitations for an antenna with specified HPBW of 20° and FNBW of 40° (N=12).

Weighting Factors				a_n				
α_1	α_2	α_3	a_0	a_1	a_2	a_3	a_4	a_5
1	1	1	1	0.800	0.484	0.184	-0.017	-0.069
0.001	1	1	1	0.936	0.667	0.171	-0.150	-0.022





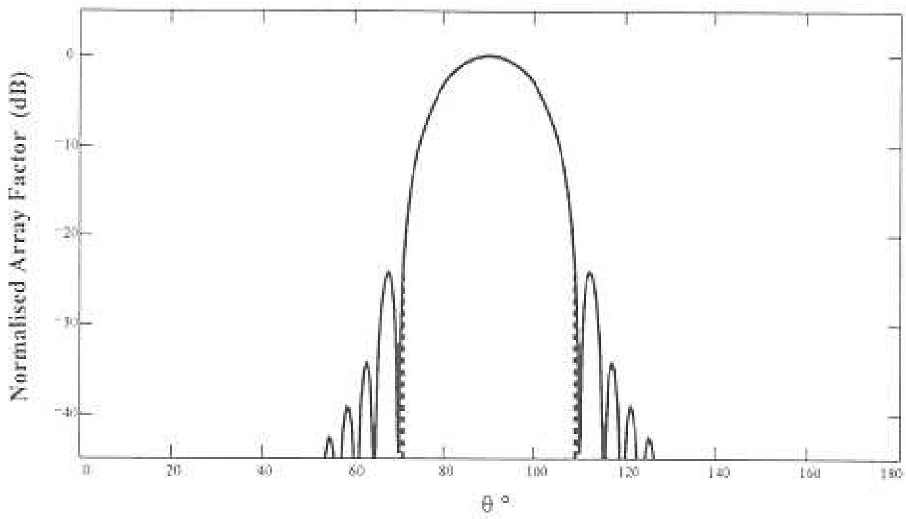


Figure 3. Synthesized radiation pattern of a linear array spaced $\lambda/2$ apart with a specified HPBW of 20° and FNBW of 40° using different weighting factors. (a) N=12, $\alpha_1=\alpha_2=\alpha_3$ (b) N=12, $\alpha_1=0.001\alpha_2$ and $\alpha_3=\alpha_2$ (c) N=36 $\alpha_1=0.001\alpha_2$ and $\alpha_3=\alpha_2$. The solid curves represents the synthesized patterns and the dashed curves the desired one.

TABLE 3

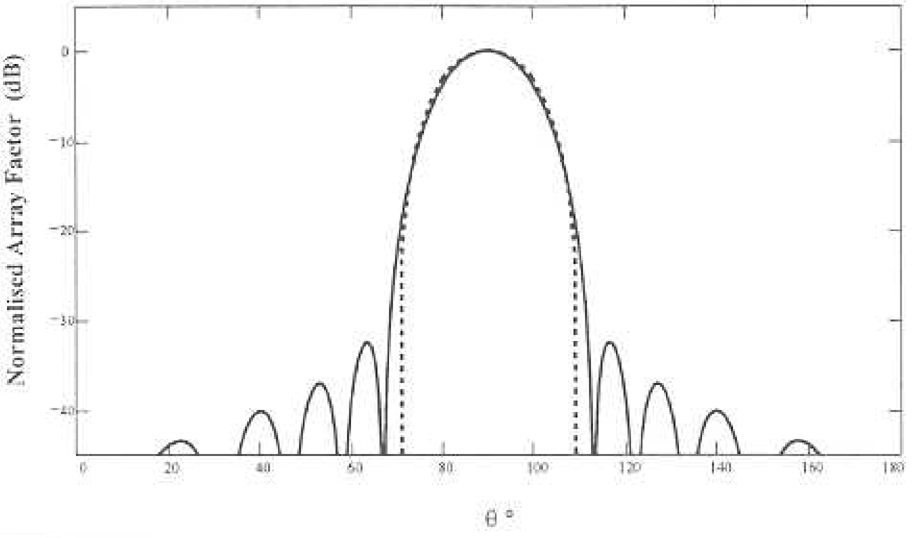
Excitations for an antenna with specified HPBW of 20° and FNBW of 40° (N=36, $\alpha_1 = 0.001\alpha_2$, $\alpha_2 = \alpha_3 = 1$).

a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
1.000	0.812	0.507	0.196	-0.016	-0.087	-0.051	-0.011	0.035
a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}
0.012	-0.018	-0.020	0.005	0.020	0.009	-0.012	-0.009	0.005

TABLE 4

Excitations for a sector antenna with specified broad nulls at 50°≤θ≤55° and 125°≤θ≤130°.

Weighting Factors				a_n					
α_0	α_1	α_2	α_3	a_0	a_1	a_2	a_3	a_4	a_5
1	1	0.1	0.1	1	0.800	0.489	0.197	0.009	-0.043
0.001	1	1	1	1	0.941	0.539	0.223	-0.086	-0.075



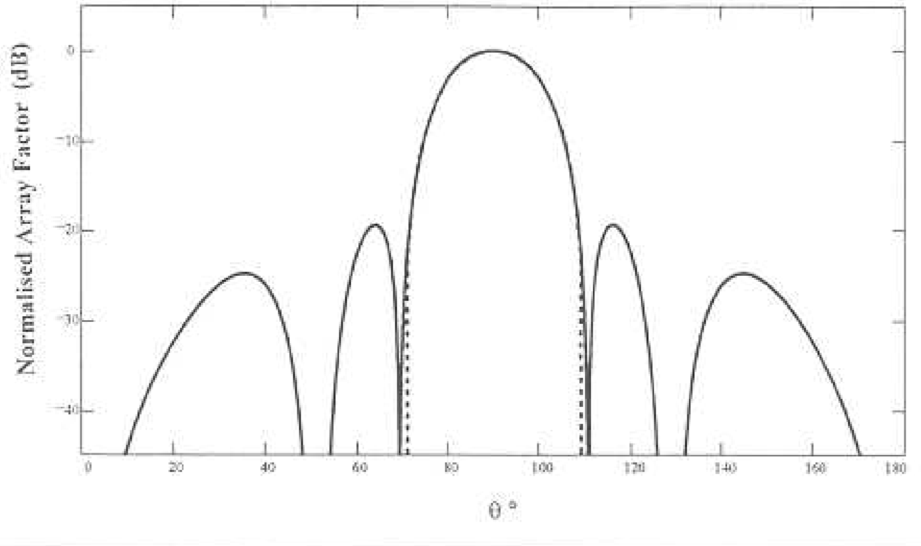


Figure 4. Synthesized radiation pattern of a 12-element array spaced λ/2 apart with a specified HPBW of 20° and FNBW of 40° and broad nulls of 5° at 50°≤θ≤55° and 125°≤θ≤130° using different weighting factors. (a) $\alpha_0=\alpha_1$ and $\alpha_2=\alpha_3=0.1\alpha_0$ (b) $\alpha_0=0.001\alpha_1$ and $\alpha_1=\alpha_2=\alpha_3$. The solid curves represent the synthesized patterns and the dashed curves the desired one.

C. ARRAY WITH SPECIFIED BROAD NULLS: An array of 12 elements is synthesized based on the following parameters: HPBW=20°, FNBW=40° and broad nulls of 5° at 50° ≤θ≤55° and 125°≤θ≤130°. D(θ) is as defined below.

Figs 4a and 4b display the synthesis results for different weighting factors. Fig.(4a) is the outcome of using $\alpha_0=\alpha_1$ and $\alpha_2=\alpha_3=0.1\ \alpha_0$ i .e. the emphasis is on the nulls and side lobe level.

$$D(\theta)=\begin{cases}\alpha_0 D_1(\theta) & 0^\circ \le \theta \le 50^\circ \text{ and } \quad 55^\circ \le \theta \le 70^\circ \\ \alpha_1 D_1(\theta) & 50^\circ \le \theta \le 55^\circ \\ \alpha_2 D_2(\theta) & 70^\circ \le \theta \le 80^\circ \\ \alpha_3 D_3(\theta) & 80^\circ \le \theta \le 100^\circ \\ \alpha_2 D_2(\theta) & 100^\circ \le \theta \le 110^\circ \\ \alpha_1 D_1(\theta) & 125^\circ \le \theta \le 130^\circ \\ \alpha_0 D_1(\theta) & 110^\circ \le \theta \le 125^\circ \text{ and } \quad 130^\circ \le \theta \le 180^\circ \end{cases}$$

where

$$D_1(\theta)=0$$

$$D_2(\theta)=\frac{\sqrt{2}}{\text{FNBW}-\text{HPBW}}\left[\frac{\text{FNBW}}{2}-\left|\theta-\frac{\pi}{2}\right|\right]$$

$$D_3(\theta)=\cos\left[\frac{\pi}{2}\left(\frac{\theta-\frac{\pi}{2}}{\text{HPBW}}\right)\right]$$

It is apparent that the main lobe is not exactly synthesized according to the design specifications although the nulls are achieved. The side lobe level here is -32 dB. If it is required to minimize the deviation between the specified pattern and the synthesised pattern, the weighting factors can be chosen to achieve that. Fig.(4b) depicts the synthesized results with α_0=0.001 and α_1=α_2=α_3=1. It is evident that the main lobe is synthesized to a high degree of accuracy and the broad nulls are at a level below -45 dB. However, these are attained at the expense of an increase in the side lobe level which is now at -19 dB (as opposed to -32 dB in Fig. (4a)). The weighting factors used and the resulting excitation coefficients are given in Table 4. This example yet again illustrates the flexibility of the proposed method.

Conclusions

A simple, non-iterative method for the synthesis of a uniformly spaced linear array with equal phase and symmetric amplitude excitation has been presented in this paper. This technique is based on formulating the synthesis process as a least-squares optimization problem. It then entails computing the least eigenvalue of a symmetric, positive definite matrix whose elements depend on the design requirements. The corresponding eigenvector yields the required element excitations. Design examples presented have shown the effectiveness and flexibility of this method in linear antenna array shaped beam synthesis.

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