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Robust Longitudinal Aircraft- Control Based on an Adaptive Fuzzy-Logic Algorithm

Abdel- Latif Elshafei

Department of Electrical Engineering, Cairo University, Giza, Egypt (Currently on leave at the United Arab Emirates).

التحكم الطولي المكين بالطائرة المعتمد على الخوارزم المنطقي المشوش (FUZZY- LOGIC)

عبداللطيف الشافعي

خلاصة : لدراسة رد الطائرة إلى مناورة سريعة ، يمكن اعتبار النموذج الطولي للطائرة لفترة زمنية قصيرة ، هذا النموذج غير خطي بدرجة كبيرة ويتضمن شكوك في قيم المعاملات . تقترح هذه الورقة استخدام نموذج قوي معتمد على الخوارزم المنطقي المشوش ، تم تنفيذ نظامان معتمدين على هذا الخوارزم وقد كشفت نتائج المحاكاة على نموذج طائرة "F16" عن نجاح النموذج المستخدم.

ABSTRACT: To study the aircraft response to a fast pull-up manoeuvre, a short period approximation of the longitudinal model is considered. The model is highly nonlinear and includes parametric uncertainties. To cope with a wide range of command signals, a robust adaptive fuzzy logic controller is proposed. The proposed controller adopts a dynamic inversion approach. Since feedback linearization is practically imperfect, robustifying and adaptive components are included in the control law to compensate for modeling errors and achieve acceptable tracking errors. Two fuzzy systems are implemented. The first system models the nominal values of the system's nonlinearity. The second system is an adaptive one that compensates for modeling errors. The derivation of the control law based on a dynamic game approach is given in detail. Stability of the closed-loop control system is also verified. Simulation results based on an F16-model illustrate a successful tracking performance of the proposed controller.

KEYWORDS: Adaptive Control, Fuzzy Logic Control, Robust Control, Flight Control.

1. Introduction

Historically, the trend in the flight control industry has been to use classical techniques for control design (Nelson 1998). Acceptable performance, simple control structure, and moderate computational burden are the reasons for adopting classical control techniques. The approach is to design several point controllers throughout the operating region and connect them using gain scheduling (Adams, *et al* 1994). Interpolation or blending point controllers we often use trial and error with little theoretical guidance. Any performance and robustness guarantees in the individual operating regions are lost in the transition region between point controllers (Spillman 2000). Dynamic inversion methods avoid the scheduling problem via feedback linearization (Adams, *et al* 1994). Like gain scheduling, dynamic inversion does not guarantee performance and robustness since cancellation is practically imperfect.

To enhance the robustness of the inverse flight controller, a design based on μ synthesis is proposed by Reiner *et al* (1995). The design utilizes a linearized model of the aircraft. Therefore, it is useful for small uncertainty in the system parameters. A fixed H_∞ controller is proposed by Chaing *et al* (1990) for a fighter aircraft with multiple control efforts. One condition along the manoeuvre trajectory is chosen as nominal and several other conditions along the manoeuvre trajectory represent the uncertainty for which the robust controller is designed. Sliding mode control is another approach that is suggested by Hedrick and Gopalswamy (1990) to achieve a high $g-$command and satisfy flying quality specifications. However, control saturation significantly alters the performance for a high $g-$command.

To take into account the relation between real-time parameter variations and performance requirements, linear parameter varying (LPV) control is examined by Spillman (2000) to determine whether it is practical for large envelop flight control designs. The approach is combined with μ synthesis to ease conservatism. The method is based on linear matrix inequalities and can be solved using the interior point method (Boyd *et al* 1994). The proposed controller does not allow parameters' rates to be modeled nor does it allow the locations of the controller poles to be constrained.

A robust adaptive controller is proposed by Singh and Steinberg (1996) as an alternative approach that ensures stability in the presence of parametric uncertainty. To derive the control law, a hypersurface is designed such that for any trajectory evolving on this surface, the system tracking error tends to zero. The objective of the control law is to drive the system error to the required hyper-surface. However, the derivation assumes that the unknown nonlinear terms depend linearly on the parameters to be estimated. Recently, an adaptive fuzzy logic algorithm was proposed for flight control systems (Wilson, 2000). An inner loop controller is designed based on a linearized aircraft model. Then, an outer-loop controller is employed based on fuzzy logic.

We propose here a robust adaptive fuzzy-logic algorithm for flight control during a fast pull-up manoeavre. The control law is based on feedback linearization. Since feedback linearization can hardly be exact, the control law is augmented to include adaptive and robustifying components so that the system can cope with modeling uncertainties and achieve acceptable tracking. In section 2, an F-16 short-period approximation of the longitudinal model is introduced. The need for a robust adaptive fuzzy-logic controller is discussed. In section 3, adaptive fuzzy-logic control is reviewed. Although it does not guarantee robustness, it is used to develop a fuzzy model for the nominal nonlinearity of the system. The estimate of the nominal nonlinearity is used in the control law of section 4 for feedback linearization. A complete derivation of the proposed control law is presented in section 4. In section 5, the implementation details and simulation results are depicted. Section 6 concludes the paper.

2. Modeling equations and design objectives

The aircraft motions can be classified as lateral and longitudinal motion (Nelson 1998). The rolling and yawing of the aircraft characterize the lateral motion. In the longitudinal mode, one assumes that the motion is confined in the vertical plane. Our interest here is directed to the $g-$command, a fast pull-up manoeavre that takes place in the vertical plane. Hence, we focus on the longitudinal dynamics. The phugoid and the short period modes characterize the longitudinal dynamics of an aircraft. The phugoid period is an order or two longer than the short period mode. To study the aircraft response to the $g-$command, it is sufficient to consider a short period approximation of the longitudinal dynamics. The required model is derived by assuming that the aircraft horizontal velocity $\boldsymbol{U}$ remains constant and by dropping the pitch angle from the states.

The short-period approximation of the longitudinal model, referred to the aircraft body frame, is summarized in Lee and Hedrick (1994) as

$$\dot{\alpha} = -\frac{(L\cos\alpha + D\sin\alpha)}{mU}\cos^2\alpha + q\cos^2\alpha \tag{1}$$

$$\dot{q} = \frac{M}{I_{yy}} \tag{2}$$

$$\delta\dot{e} = -k_e\delta e + k_e u \tag{3}$$

α is the angle of attack, q is the pitch rate, δe is the elevator angle, and u is the control signal. The angle of attack is defined as $\tan\alpha = \frac{W}{U}$, where W is the velocity along the $z-$axis of the aircraft body frame, U is the velocity along the $x-$axis of the aircraft body frame and I_{yy} is the pitch moment of inertia. The aerodynamic forces and moments D, M, and L are defined as

$$D = \bar{q}s(c_{d\alpha}\alpha + c_{d\delta e}\delta e) \tag{4}$$

$$L = \bar{q}s\left(c_{L\alpha}\alpha + \frac{c_{Lq}c}{V_t}q + c_{L\delta e}\delta e\right) \tag{5}$$

$$M = \bar{q}sc\left(c_{m\alpha}\alpha + \frac{c_{mq}c}{2V_t}q + c_{m\delta e}\delta e\right) \tag{6}$$

V_t is the aircraft speed, $\bar{q}$ is the dynamic pressure, and the coefficients c_{ii} are responsible for the lift, drag, and pitch moment of the aircraft. The definitions and typical numerical values of the variables and parameters used in (1)-(6) are given in Appendix 1.

The output $y(t)$ is the normal acceleration felt at the pilot's position.

$$y(t) = A_n + \frac{l_x}{g}\dot{q} \tag{7}$$

$$A_n = \frac{L\cos\alpha + D\sin\alpha}{mg} \tag{8}$$

A_n is the acceleration at the center of gravity of the aircraft. Equations (1), (2), (3), and (7) can be written as

$$\dot{x} = \underline{f}(\underline{x}) + \underline{b}u \tag{9}$$

$$y(t) = h(\underline{x}) \tag{10}$$

where $\underline{x} = [\alpha \quad q \quad \delta e]^T \in R^3$, $u \in R$, $y \in R$, and $\underline{b} = [0 \quad 0 \quad k_e]^T$
Differentiating (10) once yields

$$\dot{y}(t) = \Delta(\underline{x}) + \beta(\underline{x})u \tag{11}$$

Define $\Delta(\underline{x})$ and $\beta(\underline{x})$ to be

$$\Delta(\underline{x}) = \frac{\partial h(\underline{x})}{\partial \underline{x}} f(\underline{x}) \tag{12}$$

$$\beta(\underline{x}) = \frac{\partial h(\underline{x})}{\partial \underline{x}} \underline{b} \tag{13}$$

It is straightforward to show that $\beta(\underline{x})$ is given by

$$\beta(\underline{x}) = k_e\frac{\bar{q}s}{mg}[c_{L\delta e}\cos\alpha + c_{d\delta e}\sin\alpha] + k_e\frac{l_x}{g}\frac{\bar{q}sc}{I_{yy}}c_{m\delta e} \tag{14}$$

As shown in Lee and Hedrick (1994), $\beta(\underline{x})$ is non-zero. Hence, the nonlinear system (9)-(10) has a relative degree equal to one and admits feedback linearization. Choose the control law as

$$u = \frac{1}{\beta(\underline{x})}[-\Delta(\underline{x}) + v] \tag{15}$$

We select v such that the output $y(t)$ would track a reference trajectory y_d. This is achieved by

$$v = \dot{y}_d - ke \tag{16}$$

In the ideal case, the positive constant k determines the location of the closed loop pole of the error model. The error signal is defined as

$$e = y - y_d \tag{17}$$

The reference signal y_d is assumed to be smooth such that its derivative $\dot{y}_d$ exists.

To adapt to various flying conditions, the nonlinear functions $\Delta(\underline{x})$ and $\beta(\underline{x})$ can be estimated on-line. Fuzzy logic provides an attractive technique to represent such non-linearity. The power of fuzzy models stems from the universal approximation theorem (Kosko 1997). From the implementation point of view, adaptive fuzzy systems are attractive since they depend linearly on the parameters to be estimated. In section 3, an adaptive fuzzy-logic controller is derived. The control law becomes

$$u = \frac{1}{\hat{\beta}(\underline{x})}\left[-\hat{\Delta}(\underline{x}) + v\right] \tag{18}$$

$$\hat{\Delta}(\underline{x}) = \underline{\hat{\theta}}_{\Delta}^{T}\underline{\zeta}(\underline{x}) \tag{19}$$

$$\hat{\beta}(\underline{x}) = \underline{\hat{\theta}}_{\beta}^{T}\underline{\zeta}(\underline{x}) \tag{20}$$

where $\underline{\zeta}$ is the vector of fuzzy basis functions to be defined later, $\underline{\hat{\theta}}_{\Delta}$ is the vector of estimated parameters used to model $\Delta(\underline{x})$, and $\underline{\hat{\theta}}_{\beta}$ is the vector of estimated parameters used to model $\beta(\underline{x})$. According to the universal approximation theorem (Wang 1994), there exist fuzzy systems that approximate the functions $\Delta(\underline{x})$ and $\beta(\underline{x})$ with arbitrary accuracy. However, to avoid the rule explosion phenomenon, the size of $\underline{\zeta}$ is kept small. This helps in reducing the rule base and lightening the computational burden but introduces modeling errors and raises the robustness issues. In section 4, we redesign the control law such that the effect of modeling error is accommodated and compensated for.

3. Adaptive fuzzy-logic control of the longitudinal motion

In this section, we design an indirect adaptive algorithm to control the aircraft acceleration so that it tracks a given $g-$command. The control law is given in (18). As pointed out earlier, the estimates $\hat{\Delta}$ and $\hat{\beta}$ will have modeling errors when they are compared with their true values Δ and β. In Wang (1994), a supervisory controller is added to the control law to ensure robustness. The supervisory controller utilizes a **sign** function and may lead to chattering so it is not used here. In this paper, we will use the estimates $\hat{\Delta}$ and $\hat{\beta}$ as nominal values of Δ_o and β_o. In the coming section, a robust adaptive controller is redesigned based on Δ_o and β_o.

Consider the T-S fuzzy system with center average defuzzification. The fuzzy systems are used to model the nonlinear functions Δ and β. Assume for example that Δ is modeled using M rules that are denoted as $R^1, R^2, \cdots, R^M$. The i$^{\text{th}}$ rule takes the form R^i if x_1 is F_1^i and x_2 is F_2^i and x_3 is F_3^i then Δ is θ_i.

The linguistic variables x_1, x_2, and x_3 correspond to the state variables α, q, and δe, respectively. Each linguistic variable x_j is assigned a fuzzy set F_j^i that is defined using a guassian membership function $\mu_{F_j^i}$; $j = 1,2,3$. Let x_j belong to the universe of discourse $U_j \subset R$. The membership function $\mu_{F_j^i}$ maps U_j to the set $[0,1]$. The consequent of the ith rule is assigned the singleton value θ_i. The function Δ is modeled as

$$\Delta(\underline{x}) = \frac{\sum_{i=1}^{M} \theta_i \mu_i}{\sum_{i=1}^{M} \mu_i} \tag{21}$$

where μ_i is the strength of the i^{th} rule when it is fired and is calculated as

$$\mu_i = \prod_{j=1}^{3} \mu_{F_j^i} \tag{22}$$

It is assumed that the fuzzy system is constructed such that $0 \le \mu_i \le 1$ and $\sum_{i=1}^{M} \mu_i \ne 0$ for all $x_j \in U_j, j = 1,2,3$. Equation (21) can be written as

$$\Delta(\underline{x}) = \underline{\theta}_{\Delta}^{T} \underline{\zeta}(\underline{x}) \tag{23}$$

where

$$\begin{aligned} \underline{\theta}_{\Delta}^{T} &= [\theta_1 \ \cdots \ \theta_i \ \cdots \ \theta_M] \\ \underline{\zeta}^{T}(\underline{x}) &= [\zeta_1 \ \cdots \ \zeta_i \ \cdots \ \zeta_M] \\ \zeta_i &= \frac{\mu_i}{\sum_{i=1}^{M} \mu_i}, \qquad i = 1, \cdots, M \end{aligned}$$

The functions $\zeta_i, i = 1, \cdots, M$, are called the fuzzy basis functions. In an adaptive system, the values $\theta_i, i = 1, \cdots, M$,, are tuned on-line to ensure the fuzzy model is close enough to match the actual system. An expression similar to (23) can model the nonlinear function β.

It follows from (11) and (17) that

$$\dot{e} = \dot{y} - \dot{y}_d = \Delta(\underline{x}) + \beta(\underline{x})u - \dot{y}_d \tag{24}$$

Using (16) and (18), it is possible to write $\dot{y}_d$ as

$$\dot{y}_d = \hat{\Delta}(\underline{x}) + \hat{\beta}(\underline{x})u + ke \tag{25}$$

Substituting (25) into (24), the error model can be expressed as

$$\dot{e} = \tilde{\Delta}(\underline{x}) + \tilde{\beta}(\underline{x})u - ke \tag{26}$$

The error functions $\tilde{\Delta}$ and $\tilde{\beta}$ are defined as

$$\begin{aligned} \tilde{\theta}_{\Delta} &= \underline{\theta}_{\Delta} - \underline{\hat{\theta}}_{\Delta} \\ \tilde{\theta}_{\beta} &= \underline{\theta}_{\beta} - \underline{\hat{\theta}}_{\beta} \end{aligned}$$

Based on the universal approximation theorem, there are fuzzy systems Δ^* and β^* that can approximate Δ and β with arbitrary degree of accuracy. Hence, it is possible to write

$$\Delta(\underline{x}) \approx \Delta^*(\underline{x}) = \underline{\theta}_{\Delta}^{T} \underline{\zeta}(\underline{x}) \tag{27}$$

$$\beta(\underline{x}) \approx \beta^*(\underline{x}) = \underline{\theta}_{\beta}^{T} \underline{\zeta}(\underline{x}) \tag{28}$$

Using (19), (20), (27), and (28), the error model (26) becomes

$$\dot{e} = \underline{\tilde{\theta}}_{\Delta}^{T} \underline{\zeta}(\underline{x}) + \underline{\tilde{\theta}}_{\beta}^{T} \underline{\zeta}(\underline{x}) - ke \tag{29}$$

The estimation errors, $\underline{\tilde{\theta}}_{\Delta}$ and $\underline{\tilde{\theta}}_{\beta}$, are defined as

$$\begin{aligned} \underline{\tilde{\theta}}_{\Delta} &= \underline{\theta}_{\Delta} - \underline{\hat{\theta}}_{\Delta} \\ \underline{\tilde{\theta}}_{\beta} &= \underline{\theta}_{\beta} - \underline{\hat{\theta}}_{\beta} \end{aligned}$$

To derive the adaptation laws of $\underline{\tilde{\theta}}_{\Delta}$ and $\underline{\tilde{\theta}}_{\beta}$, consider the candidate Lyapunov function

$$V = \frac{1}{2}pe^2 + \frac{1}{2}\underline{\tilde{\theta}}_\Delta \Gamma_\Delta \underline{\tilde{\theta}}_\Delta + \frac{1}{2}\underline{\tilde{\theta}}_\beta \Gamma_\beta \underline{\tilde{\theta}}_\beta \tag{30}$$

The weighting factor, p, and the weighting matrices, Γ_Δ and Γ_β, are positive definite. The time derivative of (30) along the trajectory (29) is

$$\dot{V} = -pke^2 + \underline{\tilde{\theta}}_\Delta^T \underline{\zeta}(\underline{x})pe + \underline{\tilde{\theta}}_\beta^T \underline{\zeta}(\underline{x})peu + \underline{\tilde{\theta}}_\Delta^T \Gamma_\Delta \dot{\underline{\tilde{\theta}}}_\Delta + \underline{\tilde{\theta}}_\beta^T \Gamma_\beta \dot{\underline{\tilde{\theta}}}_\beta \tag{31}$$

The adaptation laws are chosen as

$$\dot{\underline{\tilde{\theta}}}_\Delta = -\Gamma_\Delta^{-1}\underline{\zeta}(\underline{x})pe \tag{32}$$

$$\dot{\underline{\tilde{\theta}}}_\beta = -\Gamma_\beta^{-1}\underline{\zeta}(\underline{x})peu \tag{33}$$

Equations (32) and (33) force the right hand side of (31) to be negative definite. Hence, equation (30) becomes a true Lyapunov function and the error model (29) is asymptotically stable. Although it is possible to argue that adaptive fuzzy logic control ensures that $e(t)$ will converge to zero, we have to remember that the above discussion overlooks the modeling errors $(\Delta - \Delta^*)$ and $(\beta - \beta^*)$. These modeling errors are inherent in fuzzy models because of the limitations on the sizes of the rule bases. In Wang (1994), a supervisory control signal is added to the adaptive fuzzy controller to ensure stability. However, the supervisory control signal is implemented using a $\operatorname{sgn}(.)$ function and may lead to the well-known chattering phenomenon. This observation motivates the use of the robust adaptive fuzzy controller that is derived in section 4.

4. Robust adaptive fuzzy-logic control

Consider the input-output differential equation (11). Assume that the nominal values $\Delta_o(\underline{x})$ and $\beta_o(\underline{x})$ are available. For example, they could be provided by an expert or estimated based on an adaptive algorithm. The control law is selected as

$$u = \frac{1}{\beta_o(\underline{x})}\left[-\Delta_o(\underline{x}) + v_o\right] \tag{34}$$

The control signal v_o is defined below. Its objectives are to ensure tracking of the desired output trajectory and robustness in the presence of modeling errors. Substituting (34) into (11) leads to

$$\dot{y} = \Delta(\underline{x}) + \left(\frac{\beta(\underline{x})}{\beta_o(\underline{x})} - 1\right)v_o + v_o - \frac{\beta(\underline{x})}{\beta_o(\underline{x})}\Delta_o(\underline{x}) \tag{35}$$

Define v_o and γ as follows

$$v_o = \dot{y}_d - ke + u_o \tag{36}$$

$$\gamma = \Delta(\underline{x}) + \left(\frac{\beta(\underline{x})}{\beta_o(\underline{x})} - 1\right)v_o - \frac{\beta(\underline{x})}{\beta_o(\underline{x})}\Delta_o(\underline{x}) \tag{37}$$

The control signal u_o, defined below, consists of two components; an adaptive fuzzy component and a robustifying component. Substituting (36) and (37) into (35), it is possible to write the system error model as

$$\dot{e} = -ke + \gamma + u_o \tag{38}$$

Let γ^* be a fuzzy system that would approximate γ with an acceptable accuracy ε, i.e.

$$\varepsilon_\gamma = \gamma - \gamma^*, \qquad |\varepsilon_\gamma| < \varepsilon \tag{39}$$

The fuzzy system γ^* is defined as

$$\gamma^* = \underline{\theta}_\gamma^{*T} \underline{\zeta}(\underline{x}, y_d, \dot{y}_d) \tag{40}$$

where $\underline{\theta}_\gamma^*$ is the optimal parameter vector that satisfies (39) and $\underline{\zeta}_\gamma(.)$ is the vector of fuzzy basis functions. The dependency of $\underline{\zeta}_\gamma$ on $\underline{x}, y_d$, and $\dot{y}_d$ follows from (36), (37), and (49). In the special case where $\beta_o = \beta$, the basis functions $\underline{\zeta}_\gamma$ depend on $\underline{x}$ only; see (37). It is possible to rewrite (38) as

$$\dot{e} = -ke + \varepsilon_\gamma + u_o + \gamma^* \tag{41}$$

The control component u_o is designed such that it cancels the effect of the modeling error γ^* and ensures robustness in the presence of ε_γ. Let u_o be

$$u_o = -\hat{\underline{\theta}}_\gamma^T \underline{\zeta}_\gamma(\underline{x}, y_d, \dot{y}_d) + u_e \tag{42}$$

where $\hat{\underline{\theta}}_\gamma$ is the estimate of $\underline{\theta}_\gamma^*$ and u_e is the robustifying component to be defined below. Equation (41) can be rewritten as

$$\dot{e} = -ke + \tilde{\underline{\theta}}_\gamma^T \underline{\zeta}_\gamma + u_e + \varepsilon_\gamma \tag{43}$$

where $\tilde{\underline{\theta}}_\gamma = \underline{\theta}_\gamma^* - \hat{\underline{\theta}}_\gamma$

Noting that ε_γ acts as a disturbance applied to the error model (43), the calculations of $\hat{\underline{\theta}}_\gamma$ and u_e will be based on a dynamic game approach (Chen *et al* 1998). The objective is to find the optimal control law u_e that minimizes a performance index, J, in the presence of the worst-case disturbance $\varepsilon_\gamma \in L_2[0, t_f]$. Consider the following minimax problem

$$\min_{u_e \in L_2[0,t_f]} \max_{\varepsilon_\gamma \in L_2[0,t_f]} \int_0^{t_f} \left(qe^2 + ru_e^2 - \rho\varepsilon_\gamma^2\right) dt$$

Define the performance index J as

$$J = \int_0^{t_f} \left(qe^2 + ru_e^2 - \rho^2 \varepsilon_\gamma^2\right) dt \tag{44}$$

where q, r, and ρ are positive weighting factors to be chosen by the designer and they have a standard interpretation in the optimal control literature. Equation (44) can be rewritten as

$$\begin{aligned} J = {} & pe^2(0) - pe^2(t_f) + \frac{1}{\sigma}\tilde{\underline{\theta}}_\gamma^T(0)\tilde{\underline{\theta}}_\gamma(0) - \frac{1}{\sigma}\tilde{\underline{\theta}}_\gamma^T(t_f)\tilde{\underline{\theta}}_\gamma(t_f) \\ & + \int_0^{t_f} qe^2(t) + ru_e^2(t) - \rho^2\varepsilon_\gamma^2(t) + \frac{d}{dt}\left(pe^2(t) + \frac{1}{\sigma}\tilde{\underline{\theta}}_\gamma^T(t)\tilde{\underline{\theta}}_\gamma(t)\right) dt \end{aligned} \tag{45}$$

Carrying out the derivative inside the integral sign and substituting for $\dot{e}(t)$ from (43), we can rewrite (45) as

$$\begin{aligned} J = {} & pe^2(0) - pe^2(t_f) + \frac{1}{\sigma}\tilde{\underline{\theta}}_\gamma^T(0)\tilde{\underline{\theta}}_\gamma(0) - \frac{1}{\sigma}\tilde{\underline{\theta}}_\gamma^T(t_f)\tilde{\underline{\theta}}_\gamma(t_f) \\ & + \int_0^{t_f} (q - 2pk)e^2(t) + ru_e^2(t) - \rho^2\varepsilon_\gamma^2(t) + 2pu_e(t)e(t) \\ & + 2p\varepsilon_\gamma(t)e(t) + 2p\tilde{\underline{\theta}}_\gamma^T(t)\underline{\zeta}_\gamma(t)e(t) + \frac{2}{\sigma}\tilde{\underline{\theta}}_\gamma^T(t)\dot{\tilde{\underline{\theta}}}_\gamma(t)dt \end{aligned} \tag{46}$$

By completing the squares, it is possible to rearrange (46) as

$$J = pe^2(0) - pe^2(t_f) + \frac{1}{\sigma}\underline{\tilde{\theta}}_\gamma^T(0)\underline{\tilde{\theta}}_\gamma(0) - \frac{1}{\sigma}\underline{\tilde{\theta}}_\gamma^T(t_f)\underline{\tilde{\theta}}_\gamma(t_f)$$
$$+\int_0^{t_f}\left(\left(q - 2pk + p^2\left(\frac{1}{r} - \frac{1}{\rho^2}\right)\right)e^2(t) + \frac{1}{r}(ru_e(t) + pe(t))^2\right.$$
$$\left. + \left(\rho\varepsilon_\gamma(t) - \frac{1}{\rho}pe(t)\right)^2 + 2\underline{\tilde{\theta}}_\gamma^T\left(p\underline{\zeta}_\gamma(t)e(t) + \frac{1}{\sigma}\dot{\underline{\tilde{\theta}}}_\gamma(t)\right)\right)dt \tag{47}$$

The minimax problem is achieved by selecting

$$q - 2pk + p^2\left(\frac{1}{r} - \frac{1}{\rho^2}\right) = 0 \tag{48}$$

$$u_e = -\frac{p}{r}e(t) \tag{49}$$

$$\dot{\underline{\tilde{\theta}}}_\gamma = -\sigma p\underline{\zeta}_\gamma(t)e(t) \tag{50}$$

The optimal control law (49) guarantees the worst-case error to be

$$e(t) = \frac{\rho^2}{p}\varepsilon_\gamma(t) \tag{51}$$

It follows from (39) and (51) that $e(t)$ is finite since $|\varepsilon_\gamma(t)|$ is bounded by ε. The error, $e(t)$, can be made smaller by decreasing ρ. On the other hand, r must be chosen such that $\frac{1}{r} \geq \frac{1}{\rho^2}$ to ensure that (48) has a positive definite solution, p Hence, if ρ is decreased, r must also be decreased which may lead to excessive control actions.

In order to further investigate the stability of the closed-loop control system, consider the following candidate Lyapunov function

$$V = \frac{1}{2}e^2 + \frac{1}{2\sigma p}\underline{\tilde{\theta}}_\gamma^T\underline{\tilde{\theta}}_\gamma \tag{52}$$

The time derivative of (52) along the trajectory (43) is

$$\dot{V} = e\dot{e} + \frac{1}{\sigma p}\underline{\tilde{\theta}}_\gamma^T\dot{\underline{\tilde{\theta}}}_\gamma$$
$$= -ke^2 + \underline{\tilde{\theta}}_\gamma^T\underline{\zeta}_\gamma e + u_e e + \varepsilon_\gamma e + \frac{1}{\sigma p}\underline{\tilde{\theta}}_\gamma^T\dot{\underline{\tilde{\theta}}}_\gamma \tag{53}$$

Using (49)-(51), it is possible to rewrite (53) as

$$\dot{V} = -\frac{\rho^4}{p}\varepsilon_\gamma^2\left(\frac{k}{p} + \frac{1}{r} - \frac{1}{\rho^2}\right) \tag{54}$$

It is clear that the right hand side of (54) is negative definite provided that $k > 0$, $p > 0$, and $\frac{1}{r} \geq \frac{1}{\rho^2}$. All the previous conditions can be satisfied since k, p, r, and ρ are the designer's choice. So, we conclude that the proposed control algorithm stabilizes the aircraft error model (43). The implementation details and some simulation results of the proposed controller are given in section 5.

5. Implementation of the proposed controller

In this section, we illustrate via simulation the performance of the proposed controller. The implementation steps can be summarized as follows:

1- Obtain the nominal values Δ_o and β_o. This can be done based on an expert's knowledge or on an identification algorithm. In the present aircraft model, we assume that β_o is given by (14) and Δ_o is estimated based on the adaptive technique described in section 3.

2- Select positive values for the controller's parameters $k, r, \rho, q,$ and σ. Then, solve (48) for p. Note that we must select $\frac{1}{r} \geq \frac{1}{\rho^2}$ to ensure that the solution of (48) yields a positive definite answer.

3- Assume $\underline{\theta}_\gamma^*$ to be locally constant and use the adaptation law (50) to calculate the estimate $\hat{\underline{\theta}}_\gamma$. Practically, the projection algorithm is implemented, instead of (51), to guarantee a bounded estimate $\hat{\underline{\theta}}_\gamma$ (Wang 1994).

4- Calculate the control signal u. It follows from (34), (36), (42), and (49), that u is given by

$$u = \frac{1}{\beta_o(\underline{x})}\left[-\Delta_o(\underline{x}) + \dot{y}_d - ke - \hat{\underline{\theta}}_\gamma^T \underline{\zeta}_\gamma - \frac{p}{r}e\right] \tag{55}$$

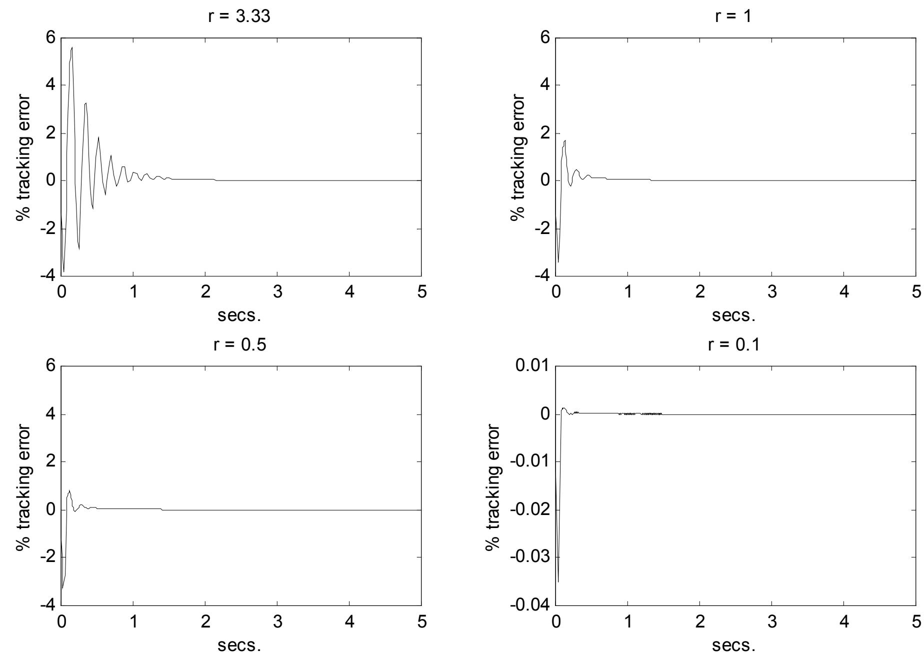

Figure 1. Tracking error performance of the proposed controller for different attenuation factors $\rho^2 = r$.

Two fuzzy systems are included to implement (55). The first fuzzy system calculates the nominal value Δ_o. The second fuzzy system is an adaptive one and is meant to compensate the function γ^*; see (41) and (42). The input to the first fuzzy system is the state vector $\underline{x}$. Each state is assigned three Guassian membership functions corresponding to the linguistic values positive, zero, and negative. All membership functions are normalized and have standard deviations 0.33. The centers of the membership functions are placed at 1, 0, and –1, respectively. The normalization factors of α, q, and δe are selected to be 0.667, 0.1, and 2, respectively. The second fuzzy system has two additional inputs; namely y_d and $\dot{y}_d$. The membership functions are similar to those used for $\underline{x}$ with the normalization factors adjusted according to the command signal. It is assumed that the nominal value $\beta_o(\underline{x})$ is 20% off the true value $\beta(\underline{x})$. The controller parameters are selected as $p = 500$, $r = \rho^2$, $\sigma = 2$, and $k = 1$. The initial values of $\hat{\underline{\theta}}_\gamma$ are initialized with random numbers in the range $[-0.05, 0.05]$. The reference trajectory, y_d, is generated via a first order system with a one-second time constant. Figure 1 depicts the performance of the proposed controller for a $5g$ command signal for different values of r. As expected, as r decreases, the tracking error decreases. However, Figure 2 shows that the cost of a very small tracking error is an unacceptably active control signal.

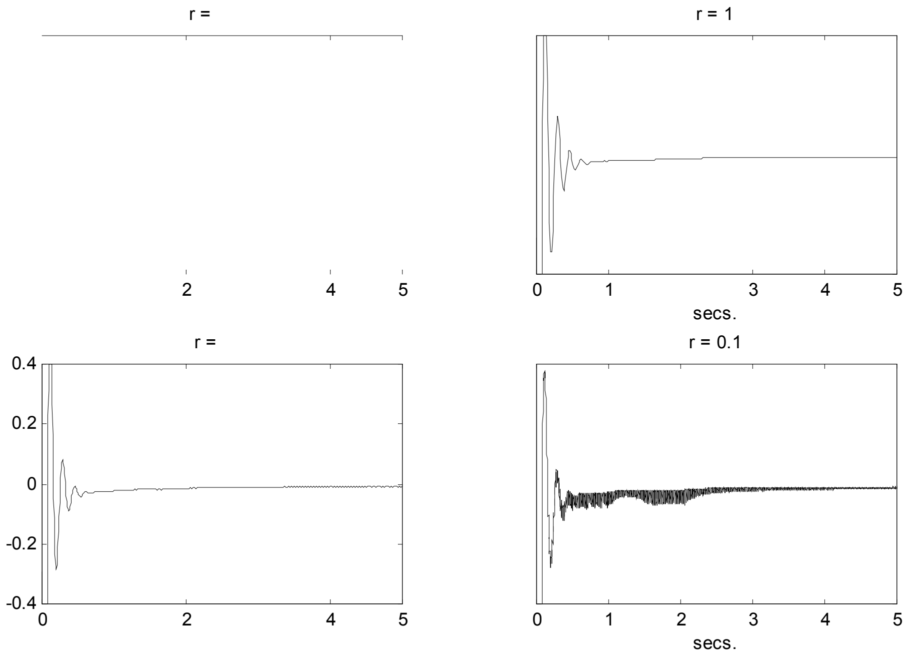

Figure 2. Control activities of the proposed controller for different attenuation factors $\rho^2 = r$.

6. Conclusions

The short-period approximation of the aircraft longitudinal model is highly nonlinear. Fuzzy logic has been used to compute the nominal values of such non-linearity. Based on the nominal values of the non-linearity, conventional feedback linearization has been modified to ensure robustness and acceptable performance. Adaptive and robustifying components have been added to the feedback linearization control law. The derivation of the proposed controller has been given in detail. It has been also shown that the tracking error has remained finite and made small using a certain tuning parameter. The stability of the proposed control system has been verified using the second method of Lyapunov. Simulation results have confirmed our theoretical analysis and demonstrated the capability of the system in tracking a high *g*-command with acceptable error and control activity.

7. Acknowledgement

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Appendix 1: Variables definitions and values at Mach 0.9 and 6096 m altitude.

Variable	Definition	Value
ρ_a	Air density	0.65381 kg/m^3
$\bar{q}$	Dynamic pressure	$\bar{q} = 0.5\rho_a V_t^2\ N/m^2$
s	Surface area	27.87899 m^2
c	Mean aerodynamic cord	3.450336 m
I_x	Distance from cg to pilot	4.244645 m
m	Mass	9530.302 kg
g	g-acceleration	9.8 $m/sec.^2$
U	Horizontal velocity	284.4 $m/sec.$
I_{yy}	Moment of inertia	73046.53 $kg\ m^2$
k_e	Elevator gain	20.0
$c_{L\alpha}$	Aerodynamic force due to α	4.0 /degree
c_{Lq}	Aerodynamic force due to q	3.162 (unitless)
$c_{L\delta e}$	Aerodynamic force due to δe	0.55 (unitless)
$c_{m\alpha}$	Aerodynamic moment due to α	0.1146 (unitless)
c_{mq}	Aerodynamic moment due to q	-2.382 (unitless)
$c_{m\delta e}$	Aerodynamic moment due to δe	-0.6933 (unitless)
$c_{d\alpha}$	Aerodynamic force due to α	0.151261 (unitless)
$c_{d\delta e}$	Aerodynamic force due to δe	0.009912 (unitless)

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