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Connected Components of the Hurwitz Space for the Symmetric Group of Degree 7

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ABSTRACT: The Hurwitz space $\mathcal{H}_{\mathrm{r}}^{\mathrm{in}}(\mathrm{G})$ is the space of genus $\mathrm{g}=0$ covers of the Riemann sphere $\mathbb{P}^1$ with r branch points and the monodromy group G. Let G be the symmetric group S_7. In this paper, we enumerate the connected components of $\mathcal{H}_{\mathrm{r}}^{\mathrm{in}}(\mathrm{S}_7)$. Our approach uses computational tools, relying on the computer algebra system GAP and the MAPCLASS package, to find the connected components of $\mathcal{H}_{\mathrm{r}}^{\mathrm{in}}(\mathrm{S}_7)$. This work gives us the complete classification of primitive genus zero symmetric group of degree seven.

Keywords: Monodromy Groups; Braid Orbits; Connected Components.

المكونات المتصلة لفضاء هيوريتز للزمرة تناظرية من الدرجة 7

هفال م. محمد صالح

الملخص: الفضاء الهوروتيزي$\mathcal{H}_{\mathrm{r,g}}^{\mathrm{A}}(\mathrm{G})$ هي نوع من غلاف في الفضاء الريماني$\mathbb{P}^1$ مع نقاط تفرع r والزمرة المونودرومية G . حيث G زمرة تناظرية S_7. في هذا البحث ، نعدد المكونات المتصلة ل $\mathcal{H}_{\mathrm{r}}^{\mathrm{in}}(\mathrm{S}_7)$. في طريقتنا نستخدم الادوات الحاسوبية بالاعتماد على نظام الجبر الحاسوبي GAP وحزمة MAPCLASS للعثور على المكونات المتصلة ب $\mathcal{H}_{\mathrm{r}}^{\mathrm{in}}(\mathrm{S}_7)$.هذا العمل يعطنا تصنيفات الجنس الصفرية للزمرة تناظرية من الدرجة السابعة.

الكلمات المفتاحية: زمرة المنودرومية، مدارات بريد و المكونات المتصلة.

1. Introduction

Let us start this section by the following definition:
A primitive genus g system is a triple $(G,\Omega,(x_1,\ldots,x_r))$ where Ω is a finite set of size n and G is a primitive subgroup of S_n such that

$$G=<x_1,\ldots,x_r> \tag{1}$$
$$\prod_{i=1}^{r} x_i=1 \tag{2}$$
$$2(n+g-1)=\sum_{i=1}^{r} ind\ x_i \tag{3}$$

where $x_i\in G\backslash\{1\}$. For $x\in G$ define $ind\ x=n-orb\ (x)$, $Fix\ x=\{w\in\Omega\ |xw=w\}$, $f(x)=|Fix\ x|$ and $orb(x)=\frac{1}{d}\sum_{i=0}^{d-1} f(x^i)$, where d is the order of x in S_n. These conditions (1), (2) and (3) are equivalent to the existence of the branched cover $\mu: X\rightarrow\mathbb{P}^1$, where X is a Riemann surface of genus g. The number of holes is called the genus, and μ is a meromorphic function where $\mathbb{P}^1=\mathbb{C}\cup\{\infty\}$ is the Riemann sphere.

Let C_i be a non-trivial conjugacy class of x_i. Then the set $C=\{C_1,\ldots,C_r\}$ in G is called the ramification type of the cover μ. Note that the trivial conjugacy class contains only the identity element.

In this paper, we classify primitive genus 0 systems for S_7. It is clear that there are seven primitive groups of degree 7. In [1], we classified all those groups except S_7. Now we are going to classify the group S_7 by using the computer algebra system GAP. All together give the complete classification of primitive genus 0 groups of degree 7.

Braid orbits can be interpreted as saying interesting things about components of the moduli space of curves $\mathcal{M}_g$ [3] and equivalence classes of branched covers of the Riemann sphere $\mathbb{P}^1$.

The full details of the following results and concepts can be found in [1-11].

Let $C_1, \dots, C_r$ be non-trivial conjugacy classes of a finite group G. The set of generating systems $(x_1, \dots, x_r)$ of G with $x_1 \dots x_r = 1$ and such that there is a permutation $\pi \in S_r$ with $x_i \in S_{\pi(i)}$ for $i = 1, \dots, r$ is called a Nielsen class and denoted by $\mathcal{N}(\mathrm{C})$, where $C = (C_1, \dots, C_r)$.

Each Nielsen class is the disjoint union of braid orbits, which are defined as the smallest subsets of the Nielsen class closed under the braid operations

$$(x_1, \dots, x_r)^{Q_i} = (x_1, \dots, x_{i+1}, x_{i+1}^{-1} x_i x_{i+1}, \dots, x_r) \tag{4}$$

for $i = 1, \dots, r$.

We denote by $\mathrm{O_r}$, the space of subsets of $\mathbb{C}$ of cardinality r.

Definition 1.1 Let $B \in O_r$ and $b_0 \in \mathbb{P}^1 \setminus B$. We call a map $\varphi: \pi_1(\mathbb{P}^1 \setminus B, b_0) \to G$ admissible if it is a surjective homomorphsim, and $\varphi(\theta_b) \neq 1$ for each $b \in B$. Here θ_b is the conjugacy class of $\pi_1(\mathbb{P}^1 \setminus B, b_0)$.

Definition 1.2 Let $B \in O_r$ and $\varphi: \pi_1(\mathbb{P}^1 \setminus B, \infty) \to G$ be admissible. Then we say that two pairs (B, φ) and $(\overline{\mathrm{B}}, \overline{\varphi})$ are A-equivalent if and only if $B = \bar{B}$ and $\bar{\varphi} = a \circ \varphi$ for some $\mathrm{a} \in \mathrm{A}$. Let $[\mathrm{B}, \varphi]_{\mathrm{A}}$ denote the A-equivalence class of (B, φ). The set of equivalence classes $[B, \varphi]_A$ is denoted by $\mathcal{H}_r^A(G)$ and is called the Hurwitz space of G-covers.

Lemma 1.3 Let C be a fixed ramifcation type in G, and the subset $\mathcal{H}_r^{in}(C)$ of $\mathcal{H}_r^{in}(G)$ consist of all $[B, \emptyset]_A$ with $B = \{b_1, \dots, b_r\}$, $\emptyset: \pi_1(\mathbb{P}^1 \backslash B, \infty) \to G$ and $\emptyset(\theta_{b_i}) \in C_i$ for $\mathrm{i} = 1, \dots, \mathrm{r}$. Then $\mathcal{H}_r^A(C)$ is a union of connected components in $\mathcal{H}_\mathrm{r}^\mathrm{A}(\mathrm{G})$. Under the bijection from Lemma 2.2, the fiber in $\mathcal{H}_r^A(C)$ over B_0 corresponds the set $\mathcal{N}^{\mathrm{A}}(\mathrm{C})$. This yields a one to one correspondence between components of $\mathcal{H}_r^A(C)$ and the braid orbits on $\mathcal{N}^A(C)$. In particular, $\mathcal{H}_r^{in}(C)$ is connected if and only if there is only one braid orbit.

Proof. For a proof see [2].

2. Computing Indexes and Labeling Conjugacy Classes

In this paper, we discuss two methods for computing index as follows:

Method one (Via Fixed Points)

Let G be a group acting on a finite set Ω of size n. If $x \in G$, define the index of x by $ind\ x = n - orb\ (x)$ where $orb\ (x)$ is the number of orbits of $< x >$ on Ω. Also $Fix\ x = \{\omega \in \Omega \mid x\omega = \omega\}$, $f(x) = |Fix\ x|$. Furthermore, $orb\ (x) = \frac{1}{d}\sum_{i=0}^{d-1} f(x^i)$ where x has order d.We discussed this method in detail in [2].

Method two (Via Cycle Types)

As we know that $ind\ x_i$ is the minimal number of 2-cycles needed to express x_i as a product. We will label the fourteen nontrivial conjugacy classes of S_7 as ATLAS notation by:

Table 1. Non trivial conjugacy classes of $\boldsymbol{S_7}$.

Type	Conjugacy class	Ind
2A	$(1,2)^{S_7}$	1
2B	$(1,2)(3,4)^{S_7}$	2
2C	$(1,2)(3,4)(5,6)^{S_7}$	3
3A	$(1,2,3)^{S_7}$	2
3B	$(1,2,3)(4,5,6)^{S_7}$	4
4A	$(1,2,3,4)^{S_7}$	3
4B	$(1,2,3,4)(5,6)^{S_7}$	4
5A	$(1,2,3,4,5)^{S_7}$	4
6A	$(1,2,3)(4,5)(6,7)^{S_7}$	4
6B	$(1,2,3)(4,5)^{S_7}$	3
6C	$(1,2,3,4,5,6)^{S_7}$	5
7A	$(1,2,3,4,5,6,7)^{S_7}$	6
10A	$(1,2,3,4,5)(6,7)^{S_7}$	5
12A	$(1,2,3,4)(5,6,7)^{S_7}$	5

3. Algorithm

To achieve connected components of $\mathcal{H}_r^{in}(G)$, we need to perform the following steps:
Step 1: Select the primitive group S_7 by using the GAP code [7] : Primitive Group (7, 7).
Step 2: Find all ramification types that satisfy equation (3) for given S_7, degree 7 and genus 0.
Step 3: Remove those types which have zero structure constant from the character table of S_7 via the following equation.

$$n(C_1, \dots, C_k) = \frac{|C_1||C_2|\dots|C_k|}{|G|} \sum_{\chi \in Irr(G)} \frac{\chi(x_1)\chi(x_2)\dots\chi(x_k)}{\chi(1)^{k-2}} \quad (5)$$

With equation (5), we compute the number of k-tuples $(x_1, \dots, x_k)$ of elements x_i in the conjugacy class C_i of a group S_7 such that $x_1 x_2 \dots x_k = 1$. In other words, we remove those types which don't satisfy equation (2).
Step 4: For the remaining types, that pass equation (1) which are called generating types.
Step 5: For the generating types, compute braid orbits by using MAPCLASS package.

Now we perform the above steps by using the program described in [12], but with a few modifications to it. That is, we remove the condition of affine type in that program. In this paper we only consider primitive groups.

4. Results

In this paper, we use the algorithm which is presented in section 3 to compute braid orbits on Nielsen class. An application of the algorithm is the classification of the primitive genus zero systems for S_7. That is, we find the connected components $\mathcal{H}_r^{in}(C)$ of S_7-curves X, such that $g(X/S\_7) = 0$. In our situation, the computation shows that there are exactly 1071 braid orbits of primitive genus 0 systems of degree 7. The degree and the number of the branch points are given in Tables 2 and 3. The detail of the Table 1 exists in [10].

Table 2. Primitive Genus Zero Systems: Number of Components.

Degree	# Group Iso types	#RTs	# comp's r = 3	# comp's r = 4	# comp's r = 5	# comp's r = 6	# comp's Total
7	5	154	179	61	67	10	317

Table 3. Primitive Genus Zero Systems: Number of Components.

Degree	# Group Iso types	#RTs	# comp's r = 3	# comp's r = 4	# comp's r = 5	# comp's r = 6	# comp's r =7,8,9,10,11,12	# comp's Total
7	1	632	171	183	172	113	61,31,14,6,2,1	754

Theorem 4.1
Up to isomorphism, there exist exactly 6 primitive genus zero groups of degree seven. The corresponding primitive genus zero groups are enumerated in Table 4 and Tables 2-3.
Lemma 4.2
The Hurwitz spaces, $\mathcal{H}_r^{in}(C)$ are connected if $G = S_7$ and $r \geq 5$.
Proof. It follows from the fact that the Nielsen classes $\mathcal{N}(C)$ are the disjoint union of braid orbits but we have only one braid orbit for S_7 and $r \geq 5$. From Lemma 1.3, we obtain that the Hurwitz spaces $\mathcal{H}_r^{in}(C)$ are connected.
Lemma 4.3
The Hurwitz spaces, $\mathcal{H}_r^{in}(C)$ are disconnected if $G = S_7$ and $r \leq 4$.
Proof. Since we have at least two braid orbits for some type C for $r \leq 4$ and $G = S_7$ and the Nielsen classes $\mathcal{N}(C)$ are the disjoint union of braid orbits. From Lemma 1.3, we obtain that the Hurwitz spaces $\mathcal{H}_r^{in}(C)$ are disconnected.

Table 4. Primitive genus zero systems of S_7.

Ramification Type	Number of orbits	Length of largest orbit	Ramification Type	Number of orbits	Length of largest orbit
(4A,5A,6C)	1	1	(3A,6B,4A,4B)	1	141
(4A,5A,10A)	2	1	(3A,6B,4A,3B)	1	64
(4A,5A,12A)	3	1	(3A,6B,4A,6A)	1	69
(4A,4B,6C)	4	1	(3A,6B,6B,5A)	1	210
(4A,4B,10A)	4	1	(3A,6B,6B,4B)	1	348
(4A,4B,12A)	4	1	(3A,6B,6B,3B)	1	168
(4A,4A,7A)	1	1	(3A,6B,6B,6A)	1	153
(4A,3B,6C)	2	1	(3A,3A,4A,6C)	1	12
(4A,3B,10A)	3	1	(3A,3A,4A,10A)	1	20
(4A,3B,12A)	1	1	(3A,3A,4A,12A)	1	18
(4A,6A,6C)	3	1	(3A,3A,6B,6C)	1	48
(4A,6A,10A)	1	1	(3A,3A,6B,10A)	1	50
(4A,6A,12A)	2	1	(3A,3A,6B,12A)	1	44
(6B,5A,6C)	6	1	(3A,3A,2C,6C)	1	20
(6B,5A,10A)	7	1	(3A,3A,2C,10A)	1	10
(6B,5A,12A)	5	1	(3A,3A,2C,12A)	1	16
(6B,4B,6C)	12	1	(3A,2C,4A,5A)	1	30
(6B,4B,10A)	9	1	(3A,2C,4A,4B)	1	42
(6B,4B,12A)	9	1	(3A,2C,4A,3B)	1	27
(6B,4A,7A)	3	1	(3A,2C,4A,6A)	1	13
(6B,3B,6C)	6	1	(3A,2C,6B,5A)	1	75
(6B,3B,10A)	4	1	(3A,2C,6B,4B)	1	105
(6B,3B,12A)	4	1	(3A,2C,6B,3B)	1	48
(6B,6A,6C)	6	1	(3A,2C,6B,6A)	1	39
(6B,6A,10A)	4	1	(3A,2C,2C,5A)	1	15
(6B,6A,12A)	3	1	(3A,2C,2C,4B)	1	32
(6B,6B,6C)	9	1	(3A,2C,2C,3B)	1	13
(3A,6C,6C)	1	1	(3A,2C,2C,6A)	1	11
(3A,10A,6C)	2	1	(2C,4A,4A,4A)	1	32
(3A,10A,10A)	1	1	(2C,6B,4A,4A)	1	88
(3A,12A,6C)	2	1	(2C,6B,6B,4A)	1	188
(3A,12A,10A)	2	1	(2C,6B,6B,6B)	2	102
(3A,12A,12A)	1	1	(2C,2C,4A,4A)	1	16
(2C,5A,6C)	3	1	(2C,2C,6B,4A)	1	56
(2C,5A,10A)	1	1	(2C,2C,6B,6B)	2	84
(2C,5A,12A)	2	1	(2C,2C,2C,4A)	1	16
(2C,4B,6C)	4	1	(2C,2C,2C,6B)	1	36
(2C,4B,10A)	3	1	(2B,4A,4A,5A)	1	40
(2C,4B,12A)	2	1	(2B,4A,4A,4B)	1	96
(2C,4A,7A)	1	1	(2B,4A,4A,3B)	1	52
(2C,3B,10A)	1	1	(2B,4A,4A,6A)	1	44
(2C,3B,12A)	1	1	(2B,6B,4A,5A)	1	150
(2C,6A,6C)	1	1	(2B,6B,4A,4B)	1	248
(2C,6A,10A)	1	1	(2B,6B,4A,3B)	1	118
(2C,6A,12A)	1	1	(2B,6B,4A,6A)	1	114
(2C,6B,7A)	3	1	(2B,6B,6B,5A)	1	395
(2B,6C,6C)	4	1	(2B,6B,6B,4B)	1	620
(2B,10A,6C)	3	1	(2B,6B,6B,3B)	1	282
(2B,10A,10A)	3	1	(2B,6B,6B,6A)	1	247
(2C,12A,6C)	3	1	(2B,3A,4A,6C)	1	33
(2C,12A,10A)	2	1	(2B,3A,4A,10A)	1	30
(2C,12A,12A)	2	1	(2B,3A,4A,12A)	1	34
(2A,6C,7A)	1	1	(2B,3A,6B,6C)	1	102
(2A,10A,7A)	1	1	(2B,3A,6B,10A)	1	85
(2A,12A,7A)	1	1	(2B,3A,6B,12A)	1	75
(4A,4A,4A,4A)	1	16	(2B,3A,2C,6C)	1	35

(6B,4A,4A,4A)	1	72	(2B,3A,2C,10A)	1	25
(6B,6B,4A,4A)	2	176	(2B,3A,2C,12A)	1	22
(6B,6B,6B,4A)	1	640	(2B,2C,4A,5A)	1	50
(6B,6B,6B,6B)	4	1008	(2B,2C,4A,4B)	1	80
(3A,4A,4A,5A)	1	15	(2B,2C,4A,3B)	1	36
(3A,4A,4A,4B)	1	44	(2B,2C,4A,6A)	1	28
(3A,4A,4A,3B)	1	23	(2B,2C,6B,5A)	1	125
(3A,4A,4A,6A)	1	34	(2B,2C,6B,4B)	1	176
(3A,6B,4A,5A)	1	65	(2B,2C,6B,3B)	1	84
(2B,2C,2C,5A)	1	35	(2B,2C,6B,6A)	1	65
(2B,2C,2C,4B)	1	48	(2A,2C,3B,4B)	1	24
(2B,2C,2C,3B)	1	22	(2A,2C,3B,3B)	1	12
(2B,2C,2C,6A)	1	18	(2A,2C,6A,5A)	1	11
(2B,2B,4A,6C)	1	72	(2A,2C,6A,4B)	1	20
(2B,2B,4A,10A)	1	60	(2A,2C,6A,3B)	1	9
(2B,2B,4A,12A)	1	52	(2A,2C,6A,6A)	1	8
(2B,2B,6B,6C)	1	198	(2A,2C,6B,6C)	1	48
(2B,2B,6B,10A)	1	140	(2A,2C,6B,10A)	1	31
(2B,2B,6B,12A)	1	124	(2A,2C,6B,12A)	1	29
(2B,2B,2C,6C)	1	60	(2A,2C,2C,6C)	1	12
(2B,2B,2C,10A)	1	40	(2A,2C,2C,10A)	1	10
(2B,2B,2C,12A)	1	36	(2A,2C,2C,12A)	1	8
(2A,4A,5A,5A)	1	12	(2A,2B,5A,6C)	1	33
(2A,4A,4B,5A)	1	39	(2A,2B,5A,10A)	1	28
(2A,4A,4B,4B)	1	80	(2A,2B,5A,12A)	1	27
(2A,4A,4A,6C)	1	12	(2A,2B,4B,6C)	1	60
(2A,4A,4A,10A)	1	16	(2A,2B,4B,10A)	1	47
(2A,4A,4A,12A)	1	20	(2A,2B,4B,12A)	1	42
(2A,4A,3B,6C)	1	23	(2A,2B,4A,7A)	1	14
(2A,4A,3B,10A)	1	38	(2A,2B,3B,6C)	1	30
(2A,4A,3B,12A)	1	12	(2A,2B,3B,10A)	1	21
(2A,4A,6A,5A)	1	26	(2A,2B,3B,12A)	1	19
(2A,4A,6A,4B)	1	34	(2A,2B,6A,6C)	1	27
(2A,4A,6A,3B)	1	21	(2A,2B,6A,10A)	1	20
(2A,4A,6A,6A)	1	10	(2A,2B,6A,12A)	1	17
(2A,6B,5A,5A)	1	62	(2A,2B,6B,7A)	1	42
(2A,6B,4B,5A)	1	119	(2A,2B,2C,7A)	1	14
(2A,6B,4B,4B)	1	188	(2A,2A,6C,6C)	1	12
(2A,6B,4A,6C)	1	48	(2A,2A,10A,6C)	1	12
(2A,6B,4A,10A)	1	49	(2A,2A,10A,10A)	1	10
(2A,6B,4A,12A)	1	41	(2A,2A,5A,7A)	1	7
(2A,6B,3B,5A)	1	54	(2A,2A,12A,6C)	1	12
(2A,6B,3B,4B)	1	90	(2A,2A,12A,10A)	1	10
(2A,6B,3B,3B)	1	42	(2A,2A,12A,12A)	1	8
(2A,6B,6A,5A)	1	59	(2A,2A,4B,7A)	1	14
(2A,6B,6A,4B)	1	77	(2A,2A,3B,7A)	1	7
(2A,6B,6A,3B)	1	36	(2A,2A,6A,7A)	1	7
(2A,6B,6A,6A)	1	28	(2B,2B,3A,6N,4A)	1	2396
(2A,6B,6B,6C)	1	144	(3A,3A,3A,4A,4A)	1	163
(2A,6B,6B,10A)	1	112	(3A,3A,3A,6B,4A)	1	606
(2A,6B,6B,12A)	1	100	(3A,3A,3A,6B,6B)	1	1827
(2A,2B,3A,2C,3B)	1	249	(2A,2A,2A,2C,7A)	1	49
(2A,2B,3A,2C,6A)	1	197	(2A,2A,3A,3A,7A)	1	49
(2A,2B,2C,4A,4A)	1	400	(2A,2A,3A,2C,6C)	1	132
(2A,2B,2C,6B,4A)	1	976	(2A,2A,3A,2C,10A)	1	85
(2A,2B,2C,6B,6B)	1	2256	(2A,2A,3A,2C,12A)	1	92
(2A,2B,2C,2C,4A)	1	276	(2A,2A,2C,4A,5A)	1	196
(2A,2B,2C,2C,6B)	1	612	(2A,2A,2C,4A,4B)	1	296
(2A,2B,2C,2C,2C)	1	168	(2A,2A,2C,4A,3B)	1	156
(2A,2B,2B,4A,5A)	1	760	(2A,2A,2C,4A,6A)	1	100
(2A,2B,2B,4A,4B)	1	1264	(2A,2A,2C,6B,5A)	1	486
(2A,2B,2B,4A,3B)	1	600	(2A,2A,2C,6B,4B)	1	684

(2A,2B,2B,4A,6A)	1	540	(2A,2A,2C,6B,3B)	1	324
(2A,2B,2B,6B,5A)	1	2115	(2A,2A,2C,6B,6A)	1	252
(2A,2B,2B,6B,4B)	1	3080	(2A,2A,2C,2C,5A)	1	120
(2A,2B,2B,6B,3B)	1	1248	(2A,2A,2C,2C,4B)	1	192
(2A,2B,2B,6B,6A)	1	1223	(2A,2A,2C,2C,3B)	1	84
(2A,2B,2B,2C,5A)	1	615	(2A,2A,2C,2C,6A)	1	72
(2A,2B,2B,2C,4B)	1	880	(2A,2A,2B,5A,5A)	1	328
(2A,2B,2B,2C,3B)	1	414	(2A,2A,2B,4B,5A)	1	592
(2A,2B,2B,2C,6A)	1	330	(2A,2A,2B,4A,6C)	1	252
(2A,2B,2B,3A,6C)	1	513	(2A,2A,2B,4B,4B)	1	952
(2A,2B,2B,3A,10A)	1	405	(2A,2A,2B,4A,10A)	1	215
(2A,2B,2B,3A,12A)	1	364	(2A,2A,2B,4A,12A)	1	206
(2A,2B,2B,2B,6C)	1	972	(2A,2A,2B,3B,5A)	1	291
(2A,2B,2B,2B,10A)	1	690	(2A,2A,2B,3B,4B)	1	444
(2A,2B,2B,2B,12A)	1	612	(2A,2A,2B,3B,3B)	1	204
(2A,2A,4A,4A,5A)	1	131	(2A,2A,2B,6B,6C)	1	720
(2A,2A,4A,4A,4B)	1	336	(2A,2A,2B,6B,10A)	1	545
(2A,2A,4A,4A,3B)	1	182	(2A,2A,2B,6B,12A)	1	487
(2A,2A,4A,4A,6A)	1	190	(2A,2A,2B,6A,5A)	1	265
(2A,2A,6B,4A,5A)	1	519	(2A,2A,2B,6A,4B)	1	388
(2A,2A,6B,4A,4B)	1	948	(2A,2A,2B,6A,3B)	1	183
(2A,2A,6B,4A,3B)	1	438	(2A,2A,2B,6A,6A)	1	148
(2A,2A,6B,4A,6A)	1	447	(2A,2A,2B,3A,7A)	1	98
(2A,2A,2B,2C,10A)	1	160	(2A,2A,2B,2C,6C)	1	228
(2A,2A,2B,2C,12A)	1	144	(2A,2A,2A,2B,4A,3B)	1	2304
(2A,2A,2B,2B,7A)	1	196	(2A,2A,2A,2B,4A,6A)	1	2076
(2A,2A,2A,5A,6C)	1	108	(2A,2A,2A,2B,6B,5A)	1	7530
(2A,2A,2A,5A,10A)	1	105	(2A,2A,2A,2B,6B,4B)	1	11796
(2A,2A,2A,5A,12A)	1	105	(2A,2A,2A,2B,6B,3B)	1	5508
(2A,2A,2A,4B,6C)	1	216	(2A,2A,2A,2B,6B,6A)	1	4818
(2A,2A,2A,4B,10A)	1	180	(2A,2A,2A,3A,6B,5A)	1	3900
(2A,3A,3A,3A,3A,4A)	1	1456	(2A,2A,2A,3A,6B,4B)	1	6720
(2A,3A,3A,3A,3A,6B)	1	4788	(2A,2A,2A,3A,6B,3B)	1	3132
(2A,3A,3A,3A,3A,2C)	1	1872	(2A,2A,2A,3A,6B,6A)	1	2991
(2A,2B,3A,3A,3A,4A)	1	3350	(2A,2A,2A,4A,4A,4A)	1	1296
(2A,2B,3A,3A,3A,6B)	1	9441	(2A,2A,2A,6B,4A,4A)	1	4428
(2A,2B,3A,3A,3A,2C)	1	2967	(2A,2A,2A,6B,6B,4A)	1	12000
(2A,2B,2B,3A,3A,4A)	1	6608	(2A,2A,2A,6B,6B,6B)	1	29802
(2A,2B,2B,3A,3A,6B)	1	17164	(2A,2A,2A,2C,4A,4A)	1	1560
(2A,2B,2B,3A,3A,2C)	1	5174	(2A,2A,2A,2C,6B,4A)	1	3828
(2A,2B,2B,2B,3A,4A)	1	12316	(2A,2A,2A,2C,6B,6B)	1	8772
(2A,2B,2B,2B,3A,6B)	1	30099	(2A,2A,2A,2C,2C,4A)	1	992
(2A,2B,2B,2B,3A,2C)	1	8658	(2A,2A,2A,2C,2C,6B)	1	2376
(2A,2B,2B,2B,2B,4A)	1	21824	(2A,2A,2A,2C,2C,2C)	1	672
(2A,2B,2B,2B,2B,6B)	1	51336	(2A,2A,2A,2B,3A,6C)	1	1836
(2A,3A,5A,6C)	1	12	(3A,3A,3A,2C,4A)	1	272
(2A,3A,5A,10A)	1	16	(3A,3A,3A,2C,6B)	1	612
(2A,3A,5A,12A)	1	17	(3A,3A,3A,2C,2C)	1	133
(2A,3A,4B,6C)	1	30	(2B,3A,3A,4A,4A)	1	418
(2A,3A,4B,10A)	1	27	(2B,3A,3A,6B,4A)	1	1285
(2A,3A,4B,12A)	1	27	(2B,3A,3A,2C,4B)	1	412
(2A,3A,4A,7A)	1	7	(2B,3A,3A,2C,6B)	1	1038
(2A,3A,3B,6C)	1	15	(2B,3A,3A,2C,2C)	1	286
(2A,3A,3B,10A)	1	16	(2B,2B,3A,4A,4A)	1	876
(2A,3A,3B,12A)	1	10	(2B,2B,3A,6B,6B)	1	5935
(2A,3A,6A,6C)	1	18	(2B,2B,3A,2C,4A)	1	748
(2A,3A,6A,10A)	1	10	(2B,2B,3A,2C,4A)	1	1721
(2A,3A,6A,12A)	1	12	(2B,2B,3A,2C,2C)	1	474
(2A,3A,6B,7A)	1	21	(2B,2B,2B,4A,4A)	1	1712
(2A,3A,2C,7A)	1	7	(2B,2B,2B,6B,4A)	1	4296

(2A,2C,5A,5A)	1	26	(2B,2B,2B,6B,6B)	1	10128
(2A,2C,4B,5A)	1	38	(2B,2B,2B,2C,4A)	1	1248
(2A,2C,4B,4B)	1	56	(2B,2B,2B,2C,6B)	1	2880
(2A,2C,4A,6C)	1	20	(2B,2B,2B,2C,2C)	1	748
(2A,2C,4A,10A)	1	11	(2A,3A,4A,4A,4A)	1	156
(2A,2C,4A,12A)	1	14	(2A,3A,6B,4A,4A)	1	589
(2A,2C,3B,5A)	1	21	(2A,3A,6B,6B,4A)	1	1782
(2A,3A,3A,6B,5A)	1	525	(2A,3A,6B,6B,6B)	1	4455
(2A,3A,3A,6B,4B)	1	996	(2A,3A,3A,4A,5A)	1	145
(2A,3A,3A,6B,3B)	1	462	(2A,3A,3A,4A,4B)	1	350
(2A,3A,3A,6B,6A)	1	468	(2A,3A,3A,4A,3B)	1	172
(2A,3A,3A,3A,6C)	1	108	(2A,3A,3A,4A,6A)	1	216
(2A,3A,3A,3A,10A)	1	135	(2A,2A,6B,6B,5A)	1	1267
(2A,3A,3A,3A,12A)	1	120	(2A,2A,6B,6B,4B)	1	2352
(2A,3A,3A,2C,5A)	1	210	(2A,2A,6B,6B,3B)	1	1098
(2A,3A,3A,2C,12A)	1	300	(2A,2A,6B,6B,6A)	1	975
(2A,3A,3A,2C,3B)	1	168	(2A,2A,3A,6A,5A)	1	170
(2A,3A,3A,2C,6A)	1	105	(2A,2A,3A,6A,4B)	1	236
(2A,3A,2C,4A,4A)	1	238	(2A,2A,3A,6A,3B)	1	126
(2A,3A,2C,6B,4A)	1	587	(2A,2A,3A,6A,6A)	1	82
(2A,3A,2C,6B,6B)	1	1335	(2A,2A,3A,5A,5A)	1	131
(2A,3A,2C,2C,4A)	1	136	(2A,2A,3A,4B,5A)	1	302
(2A,3A,2C,2C,6B)	1	366	(2A,2A,3A,4B,4B)	1	548
(2A,3A,2C,2C,2C)	1	108	(2A,2A,3A,4A,6C)	1	82
(2A,2B,4A,4A,4A)	1	384	(2A,2A,3A,4A,10A)	1	120
(2A,2B,6B,4A,4A)	1	1232	(2A,2A,3A,4A,12A)	1	123
(2A,2B,6B,6B,4A)	1	3196	(2A,2A,3A,3B,5A)	1	156
(2A,2B,3A,4A,5A)	1	355	(2A,2A,3A,3B,4B)	1	264
(2A,2B,6B,6B,6B)	1	7752	(2A,2A,3A,3B,3B)	1	105
(2A,2B,3A,4A,4B)	1	690	(2A,2A,2A,4B,12A)	1	168
(2A,2B,3A,4A,3B)	1	349	(2A,2A,2A,4A,7A)	1	49
(2A,2B,3A,4A,6A)	1	315	(2A,2A,2A,3B,6C)	1	108
(2A,2B,3A,6B,5A)	1	1085	(2A,2A,2A,3B,10A)	1	90
(2A,2B,3A,6B,4B)	1	1763	(2A,2A,3A,6B,6C)	1	360
(2A,2B,3A,6B,3B)	1	828	(2A,2A,3A,6B,6A)	1	325
(2A,2B,3A,6B,6A)	1	754	(2A,2A,3A,6B,12A)	1	286
(2A,2B,3A,3A,6C)	1	252	(2A,2A,2A,3B,12A)	1	72
(2A,2B,3A,3A,10A)	1	220	(2A,2A,2A,6A,6C)	1	108
(2A,2B,3A,3A,12A)	1	220	(2A,2A,2A,6A,10A)	1	75
(2A,2B,3A,2C,5A)	1	350	(2A,2A,2A,6A,12A)	1	7
(2A,2B,3A,2C,4B)	1	536	(2A,2A,2A,6B,7A)	1	147
(2A,2B,2B,2B,2B,2C)	1	14448	(2A,2A,2A,2B,3A,10A)	1	1500
(2A,2A,3A,3A,4A,4A)	1	1398	(2A,2A,2A,2B,3A,12A)	1	1402
(2A,2A,3A,3A,6B,4A)	1	4588	(2A,2A,2A,2B,2C,5A)	1	2340
(2A,2A,3A,3A,6B,6B)	1	12600	(2A,2A,2A,2B,2C,4B)	1	3456
(2A,2A,3A,3A,3A,5A)	1	1260	(2A,2A,2A,2B,2C,3B)	1	1620
(2A,2A,3A,3A,3A,4B)	1	2640	(2A,2A,2A,2B,2C,6A)	1	1296
(2A,2A,3A,3A,3A,3B)	1	1260	(2A,2A,2A,2B,2B,6C)	1	3564
(2A,2A,3A,3A,3A,6A)	1	1422	(2A,2A,2A,2B,2B,10A)	1	2650
(2A,2A,2B,3A,3A,5A)	1	2795	(2A,2A,2A,2B,2B,12A)	1	2327
(2A,2A,2B,3A,3A,4B)	1	4952	(2A,2A,2A,2A,4A,6C)	1	864
(2A,2A,2B,3A,3A,3B)	1	2418	(2A,2A,2A,2A,4A,10A)	1	800
(2A,2A,2B,3A,3A,6A)	1	2197	(2A,2A,2A,2A,4A,12A)	1	784
(2A,2A,2B,2B,3A,5A)	1	5530	(2A,2A,2A,2A,5A,5A)	1	1110
(2A,2A,2B,2B,3A,4B)	1	8896	(2A,2A,2A,2A,6B,6C)	1	2592
(2A,2A,2B,2B,3A,3B)	1	4191	(2A,2A,2A,2A,6B,10A)	1	2100
(2A,2A,2B,2B,3A,6A)	1	3671	(2A,2A,2A,2A,6B,12A)	1	1872
(2A,2A,3A,3A,2C,4A)	1	1688	(2A,2A,2A,2A,3A,7A)	1	343
(2A,2A,3A,3A,2C,6B)	1	4044	(2A,2A,2A,2A,2C,6C)	1	864
(2A,2A,3A,3A,2C,2C)	1	1020	(2A,2A,2A,2A,2C,10A)	1	600
(2A,2A,2B,3A,4A,4A)	1	3150	(2A,2A,2A,2A,2C,12A)	1	576
(2A,2A,2B,3A,2C,4A)	1	2824	(2A,2A,2A,2A,4A,7A)	1	686

(2A,2A,2B,3A,2C,6B)	1	6744	(2A,2A,2A,2A,4B,5A)	1	2160
(2A,2A,2B,3A,2C,2C)	1	1860	(2A,2A,2A,2A,4B,4B)	1	3648
			(2A,2A,2A,2A,6A,5A)	1	1050
(2A,2A,2B,2B,4A,4A)	1	6320	(2A,2A,2A,2A,6A,4B)	1	1536
(2A,2A,2B,2B,6B,4A)	1	11628	(2A,2A,2A,2A,6A,3B)	1	756
(2A,2A,2B,2B,6B,6B)	1	39288	(2A,2A,2A,2A,6A,6A)	1	756
(2A,2A,2B,2B,2C,4A)	1	4880	(2A,2A,2A,2A,3B,5A)	1	1080
(2A,2A,2B,2B,2C,6B)	1	11268	(2A,2A,2A,2A,3B,4B)	1	1728
(2A,2A,2B,2B,2C,2C)	1	3080	(2A,2A,2A,2A,3B,3B)	1	750
(2A,2A,2A,3A,3A,6C)	1	864	(2A,2A,2B,2B,3A,3A,3A)	1	48859
(2A,2A,2A,3A,3A,10A)	1	850	(2A,2A,2A,3A,4A,4B)	1	2516
(2A,2A,2A,3A,3A,12A)	1	808	(2A,2A,2A,3A,4A,3B)	1	1260
(2A,2A,2B,2B,2B,5A)	1	10215	(2A,2A,2A,3A,4A,6A)	1	1290
(2A,2A,2B,2B,2B,4B)	1	15408	(2A,2A,2A,3A,2C,5A)	1	1365
(2A,2A,2B,2B,2B,3B)	1	7182	(2A,2A,2A,3A,2C,4B)	1	2040
(2A,2A,2B,2B,2B,6A)	1	6066	(2A,2A,2A,3A,2C,3B)	1	1026
(2A,2A,2A,3A,4A,5A)	1	1205	(2A,2A,2A,3A,2C,6A)	1	744
(2A,2A,2A,2B,4A,4B)	1	4744	(2A,2A,2A,2B,4A,5A)	1	2710
			(2A,2A,2B,2B,2B,2B,2B)	1	260848
(2A,2A,2A,2A,2A,4A,5A)	1	9500	(2A,2A,23A,3A,3A,3A,3A)	1	12207
(2A,2A,2A,2A,2A,4A,4B)	1	17600	(2A,2A,2B,3A,3A,3A,3A)	1	25860
(2A,2A,2A,2A,2A,4A,3B)	1	8640	(2A,2A,2A,2A,2A,2A,2B,5A)	1	142500
(2A,2A,2A,2A,2A,4A,6A)	1	8160	(2A,2A,2A,2A,2A,2A,2B,4B)	1	226560
(2A,2A,2A,2A,2A,6B,5A)	1	27750	(2A,2A,2A,2A,2A,2A,2B,3B)	1	106920
(2A,2A,2A,2A,2A,6B,4B)	1	45120	(2A,2A,2A,2A,2A,2A,2B,6A)	1	92880
(2A,2A,2A,2A,2A,6B,3B)	1	21060	(2A,2A,2A,2A,2A,2A,3A,5A)	1	73125
(2A,2A,2A,2A,2A,6B,6A)	1	19080	(2A,2A,2A,2A,2A,2A,3A,4B)	1	126720
(2A,2A,2A,2A,2A,2C,5A)	1	9000	(2A,2A,2A,2A,2A,2A,3A,3B)	1	60750
			(2A,2A,2A,2A,2A,2A,3A,6A)	1	56160
(2A,2A,2A,2A,2A,2C,4B)	1	13440	(2A,2A,2A,2A,3A,3A,3A,3A)	1	92880
(2A,2A,2A,2A,2A,2C,3B)	1	8480			
(2A,2A,2A,2A,2A,2C,6A)	1	5040			
(2A,2A,2A,2A,2A,2B,6C)	1	12960	(2A,2A,2A,2A,2B,3A,3A,3A)	1	183252
(2A,2A,2A,2A,2A,2B,10A)	1	10000			
(2A,2A,2A,2A,2A,2B,12A)	1	9120	(2A,2A,2A,2A,2B,2B,3A,3A)	1	336372
(2A,2A,2A,2A,2A,2A,7A)	1	2401			
(2A,2A,2A,2A,2B,2B,5A)	1	38300	(2A,2A,2A,2A,2B,2B,2B,3A)	1	593244
(2A,2A,2A,2A,2B,2B,4B)	1	59264	(2A,2A,2A,2A,2B,2B,2B,2B)	1	1016904
			(2A,2A,2A,2A,2A,3A,3A,4A)	1	87840

Table 5. Primitive genus zero systems of $\boldsymbol{S_7}$.

(2A,2A,2A,2A,2B,2B,3B)	1	27756	(2A,2A,2A,2A,2A,3A,3A,6B)	1	244020
(2A,2A,2A,2A,2B,2B,6A)	1	23760	(2A,2A,2A,2A,2A,3A,3A,2C)	1	78000
(2A,2A,2A,2A,2B,3A,5A)	1	20250	(2A,2A,2A,2A,2A,2B,3A,4A)	1	172920
(2A,2A,2A,2A,2B,3A,4B)	1	33696	(2A,2A,2A,2A,2A,2B,3A,6B)	1	442720
(2A,2A,2A,2A,2B,3A,3B)	1	16062	(2A,2A,2A,2A,2A,2B,3A,2C)	1	131760
(2A,2A,2A,2A,2B,3A,6A)	1	14304	(2A,2A,2A,2A,2A,2B,2B,4A)	1	317920
(2A,2A,2A,2A,3A,3A,5A)	1	9825	(2A,2A,2A,2A,2A,2B,2B,6B)	1	772200
(2A,2A,2A,2A,3A,3A,4B)	1	18432	(2A,2A,2A,2A,2A,2B,2B,2C)	1	221760
(2A,2A,2A,2A,3A,3A,3B)	1	8910	(2A,2A,2A,2A,2A,2A,4A,4A)	1	83360
(2A,2A,2A,2A,3A,3A,6A)	1	8796	(2A,2A,2A,2A,2A,2A,6B,4A)	1	231840
(2A,2A,2A,2A,2A,3A,6C)	1	6480	(2A,2A,2A,2A,2A,2A,6B,6B)	1	581040
(2A,2A,2A,2A,2A,3A,10A)	1	5625	(2A,2A,2A,2A,2A,2A,2C,4A)	1	72960
(2A,2A,2A,2A,2A,3A,12A)	1	5280	(2A,2A,2A,2A,2A,2A,2C,6B)	1	172800
(2A,2A,2A,2A,2B,2C,4A)	1	18816	(2A,2A,2A,2A,2A,2A,2C,2C)	1	47040
(2A,2A,2A,2A,2B,2C,6B)	1	44064	(2A,2A,2A,2A,2A,2A,2A,6C)	1	46656
(2A,2A,2A,2A,2B,2C,2C)	1	12096	(2A,2A,2A,2A,2A,2A,2A,10A)	1	37500
(2A,2A,2A,2A,3A,2C,4A)	1	11040	(2A,2A,2A,2A,2A,2A,2A,12A)	1	43560
(2A,2A,2A,2A,3A,2C,6B)	1	26448	(2A,2A,2A,2A,2A,2A,3A,3A,3A)	1	676830
(2A,2A,2A,2A,3A,2C,2C)	1	7056	(2A,2A,2A,2A,2A,2A,2B,3A,3A)	1	1275960
(2A,2A,2A,2A,2B,4A,4A)	1	23128	(2A,2A,2A,2A,2A,2A,2B,2B,3A)	1	2283120
(2A,2A,2A,2A,2A,6B,4A)	1	61836	(2A,2A,2A,2A,2A,2A,2B,2B,2B)	1	3954720
(2A,2A,2A,2A,2B,6B,6B)	1	151428	(2A,2A,2A,2A,2A,2A,2A,3A,4A)	1	638400
(2A,2A,2A,2A,3A,4A,4A)	1	11036	(2A,2A,2A,2A,2A,2A,2A,3A,6B)	1	1682100
(2A,2A,2A,2A,3A,6B,4A)	1	33096	(2A,2A,2A,2A,2A,2A,2A,3A,2C)	1	514080
(2A,2A,2A,2A,3A,6B,6B)	1	86070	(2A,2A,2A,2A,2A,2A,2A,2B,4A)	1	1202880
(2A,2A,2A,2B,2B,2B,4A)	1	83536	(2A,2A,2A,2A,2A,2A,2A,2B,6B)	1	2978640
(2A,2A,2A,2B,2B,2B,6B)	1	82632	(2A,2A,2A,2A,2A,2A,2A,2B,2C)	1	866880
(2A,2A,2A,2B,2B,2B,2C)	1	56616	(2A,2A,2A,2A,2A,2A,2A,2A,5A)	1	525000
(2A,2A,2A,2B,2B,3A,4A)	1	46296	(2A,2A,2A,2A,2A,2A,2A,2A,10A)	1	880160
(2A,2A,2A,2B,2B,3A,6B)	1	115674	(2A,2A,2A,2A,2A,2A,2A,2A,3B)	1	408240
(2A,2A,2A,2B,2B,3A,2C)	1	33876	(2A,2A,2A,2A,2A,2A,2A,2A,6A)	1	362880
(2A,2A,2A,2B,3A,3A,4A)	1	24320	(2A,2A,2A,3A,3A,3A,4A)	1	11656
(2A,2A,2A,2B,3A,3A,6B)	1	65100	(2A,2A,2A,3A,3A,3A,6B)	1	34695
(2A,2A,2A,2B,3A,3A,2C)	1	19908	(2A,2A,2A,3A,3A,3A,2C)	1	11910
(2A,2A,2B,2B,2B,2B,3A)	1	153498	(2A,2A,2B,2B,2B,3A,3A)	1	88254

(2A,2A,2A,2A,2A,2A,2A,2A,3A,3A)	1	4798080
(2A,2A,2A,2A,2A,2A,2A,2A,2B,3A)	1	8749440
(2A,2A,2A,2A,2A,2A,2A,2A,2B,2B)	1	15355040
(2A,2A,2A,2A,2A,2A,2A,2A,2A,4A)	1	4515840
(2A,2A,2A,2A,2A,2A,2A,2A,2A,6B)	1	11430720
(2A,2A,2A,2A,2A,2A,2A,2A,2A,2C)	1	3386880
(2A,2A,2A,2A,2A,2A,2A,2A,2A,2A,3A)	1	33339600
(2A,2A,2A,2A,2A,2A,2A,2A,2A,2A,2B)	1	59270400
(2A,2A,2A,2A,2A,2A,2A,2A,2A,2A,2A,2A)	1	228191040

Conclusion

Here, we compute braid orbits on Nielsen class with the aid of the computer algebra system GAP and MAPCLASS package. A result of the algorithm is that it gives the complete classification of the symmetric group S_7 up to braid actions and diagonal conjugations. The computation shows that there are exactly 754 braid orbits of S_7. As a consequence of Lemma 1.3, we find the connected components $\mathcal{H}_r^{in}(S_7)$ of S_7-curves X, such that $g = 0$. So we have 754 connected components $\mathcal{H}_r^{in}(S_7)$ of the symmetric group of degree seven.

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