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Spectral Analysis of Magnetic Anomalies Due to a 2-D Horizontal Circular Cylinder: A Hartley Transforms Technique

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التحليل الطيفي للشواذ المغناطيسية الناتجة من اسطوانة أفقية ذو مقطع دائري ثنائية الأبعاد: تقنية محولات هارتلي

منصورعبدالله القرني

ملخص: لقد تم استخدام محول هارتلي للتحليل العمودي لشواذ المغناطيسية الناتجة من اسطوانة أفقية ذو مقطع دائري ثنائية الأبعاد. يعتبر محول هارتلي كوسيلة بديلة للتحليل الطيفي على غرار التحليل الطيفي باستخدام محول فورير. لقد تم حساب العمق الى مركز الأسطوانة الأفقية باستخدام معادلة رياضية بسيطة كدالة في التردد. كما انه تم استخدام مثال مصطنع لتوضيح خطوات هذه التقنية ومدى صلاحيتها. بالإضافة الى ذلك فإنه قد تم تطبيق هذه الطريقة المقترحة بنجاح على شاذة حقلية على شريط من الكوارتز- ماغنيتيت مأخوذة من مانغامبالي بالقرب من بلدة كاريمناغار، الهند. لقد تمت دراسة تأثير الشوشرة العشوائية على الطريقة المقترحة وأظهرت مستوى عال من الثقة. كما أن نتائج الطريقة المقترحة على مثال الشاذة الحقلية أظهرت تطابقا مع نتائج الطرق الأخرى المنشورة.

ABSTRACT: The spectral analysis of the vertical effect of magnetic anomalies due to a 2-D horizontal circular cylinder is presented using Hartley transform. Hartley transform is an alternative approach to the famous complex Fourier transform. The depth to the center of the horizontal cylinder can be computed by a simple equation as a function of frequency. A synthetic example has been used to illustrate this technique and the validity of this approach has been proved by applying it to real data of a narrow band of quartz-magnetite in Mangampalli near Karimnagar town, India. The noise analyses were tested on the technique and showed a high level of confidence. The results of the field example are in good agreement with the ones published in the literature.

KEYWORDS: Hartley transform; Magnetic; 2-D Horizontal cylinder.

1. Introduction

The Hartley transform (Hartley, 1942) has gained an importance in the field of geophysics in the last decade (Bracewell, 1983; Villasenor and Bracewell, 1987; Saatcilar *et al.* 1990; Saatcilar and Ergintav, 1991;

Sundararajan 1995, 1997; Sundararajan *et al.* 2007). The importance of this transform has been ignored not because of the complexity of the transform but because scientists have been overwhelmed by the complex algebra concept (Sundararajan *et al.* 2007).

The Hartley transform is purely real and exactly equivalent to the Fourier transform (Bracewell, 1983; Rajan, 1993; Sundararajan, 1995). The significance of this transform is that it requires no assumptions to be made, unlike the Fourier transform (Mohan and Seshagiri Rao, 1982).

The Hartley and Fourier transforms provide two numbers, having the same information at each frequency, which represent a physical oscillation in amplitude and phase.

Sundararajan and Brahmam (1998) used the Hartley transform to interpret gravity anomalies caused by slab-like structures. Sundararajan *et al.* (2007) used the Hartley transform to interpret the deformation of a homogeneous electric field over a thin bed. In this paper, the Hartley transform approach is used to estimate the causative target parameters of a 2-D horizontal circular cylinder from its magnetic anomaly. This approach is applied first to a theoretical example to illustrate the method and then applied to the vertical magnetic anomaly over a narrow band of quartz-magnetite in Mangampalli near Karimnagar town, India, to demonstrate the applicability of the method.

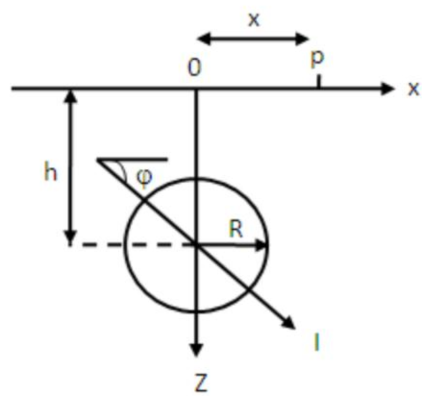

Figure 1. Geometry of the 2-D horizontal circular cylinder.

2. The Magnetic effect of a horizontal cylinder

The vertical magnetic effect of a buried horizontal circular cylinder extending infinitely in the horizontal direction along the *Y*-direction, with its normal section parallel to the *x*-*z* plane, is considered. The coordinate system origin is taken on the ground surface such that the *Z*-axis coincides with the diameter (Figure 1). Hence, the vertical magnetic effect at a point *P* on the surface can be expressed as follows (Mohan *et al.* 1982).

$$V(x) = K\left[\frac{\left(h^2 - x^2\right)\sin\varphi - 2\,x\,h\cos\varphi}{\left(x^2 + h^2\right)^2}\right] \tag{1}$$

where $K = 2\pi R^2 I, R$ is the radius of the cylinder, I is the magnetization intensity, φ is the polarization angle, and h is the depth to the center of the cylinder.

3. Hartley transform of the magnetic effect

The Hartley transform of the real function *V*(*x*) is defined by Hartley (1942) as:

$$H(\omega)=\int_{-\infty}^{\infty}V(x)\,cas(\omega x)\,dx \tag{2}$$

where

$$cas(\omega x)=\cos(\omega x)+\sin(\omega x)=\sqrt{2}\sin(\omega x+\pi/4) \tag{3}$$

is the kernel which represents a phase-shifted sine wave of 45° and hence takes the harmonics of both cosine and sine functions. The frequency ω has the same physical significance as in the case of the Fourier transform. By analogy with the real and imaginary components of the Fourier transform, the Hartley transform may be expressed in terms of its even and odd components as:

$$H(\omega)=E(\omega)+O(\omega) \tag{4}$$

where

$$E(\omega)=\int_{-\infty}^{\infty}V(x)\cos(\omega x)\,dx=\frac{H(\omega)+H(-\omega)}{2} \tag{5}$$

and

$$o(\omega)=\int_{-\infty}^{\infty}V(\omega)\sin(\omega x)\,dx=\frac{H(\omega)-H(-\omega)}{2} \tag{6}$$

where

$$H(-\omega)=\int_{-\infty}^{\infty}V(\omega)\,cas(-\omega x) \tag{7}$$

and

$$cas(-\omega x)=\cos(\omega x)-\sin(\omega x)=\sqrt{2}\sin(-\omega x+\pi/4) \tag{8}$$

Hence the amplitude and phase spectra can be expressed, respectively, as:

$$A(\omega)=\sqrt{E(\omega)^2+O(\omega)^2} \tag{9}$$

$$\vartheta(\omega)=\tan^{-1}\left[\frac{-O(\omega)}{E(\omega)}\right] \tag{10}$$

Numerically, the amplitude $A(\omega)$ is equivalent to the Fourier amplitude; however, the phase-shifted $\vartheta(\omega)$ differs by 45° from that of Fourier phase $\vartheta_F(\omega)$.

Therefore,

$$\vartheta(\omega)=\vartheta_F+\frac{\pi}{4} \tag{11}$$

Alternately, the amplitude $A(\omega)$ and phase-shifted $\vartheta(\omega)$ can be computed as:

$$A(\omega)=\sqrt{\frac{H(\omega)^2+H(-\omega)^2}{2}} \tag{12}$$

$$\vartheta(\omega)=\tan^{-1}\left[\frac{H(\omega)-H(-\omega)}{H(\omega)+H(-\omega)}\right] \tag{13}$$

Substituting for $V(x)$ in equation (1) into equation (2), the even and odd components of the Hartley transform for the vertical magnetic effect of the horizontal circular cylinder infinitely extending in the horizontal direction can be easily evaluated as:

$$E(\omega) = K\pi\omega e^{-\omega h} \sin\varphi \tag{14}$$

$$O(\omega) = K\pi\omega e^{-\omega h} \cos\varphi \tag{15}$$

Therefore, the Hartley transform $H(\omega)$ (sum of the even and odd components), amplitude $A(\omega)$ and phase-shifted $\vartheta(\omega)$ of the horizontal circular cylinder infinitely extending in the horizontal direction can be given, respectively, as:

$$H(\omega) = K\pi\omega e^{-\omega h} (\sin\varphi + \cos\varphi) \tag{16}$$

$$A_H(\omega) = K\pi\omega e^{-\omega h} \tag{17}$$

$$\vartheta = \frac{\pi}{2} - \varphi \tag{18}$$

4. Parameters evaluation

At two successive frequencies ω_i and ω_{i+1},

$$A_H(\omega_i) = K\pi\omega_i e^{-\omega_i h} \tag{19}$$

$$A_H(\omega_{i+1}) = K\pi\omega_{i+1} e^{-\omega_{i+1} h} \tag{20}$$

Where $\omega = 2\pi / N\Delta x$ is the fundamental frequency expressed in radian per unit length, N is the total number of samples and Δx is the station interval. At $i = 1$ and dividing equation (19) by equation (20), one can obtain:

$$e^{h(\omega_2 - \omega_1)} = \frac{A(\omega_1)\omega_2}{A(\omega_2)\omega_1} \tag{21}$$

Taking the natural logarithm of both sides:

$$h = \frac{1}{\omega_2 - \omega_1}\left[1\text{n}\frac{A(\omega_1)}{A(\omega_2)} + 1\text{n}\frac{\omega_2}{\omega_1} \right] \tag{22}$$

The term K is evaluated by substituting the value of h in equation (19)

$$K = \frac{A_H(\omega)}{\pi\omega} e^{\omega h} \tag{23}$$

and φ can be computed from equations (14 and 15) as:

$$\varphi = \tan^{-1}\left[\frac{E(\omega)}{O(\omega)} \right] \tag{24}$$

Therefore, based on equations (22) – (24), we can easily estimate the depth h of the polarization angle φ and the magnetization intensity related parameter K of the buried cylinder.

5. Synthetic example

The Hartley transform approach is illustrated by a synthetic model assuming a depth to the center of the horizontal circular cylinder $h = 10$ units, a polarization angle $\varphi = 60^{\circ}$ and $K = 1$ unit (Figure 2). The even component, the odd component, the Hartley transform and the amplitude spectrum are computed and shown in Figures 3a, b, c and d, respectively. Using the method that has been developed throughout the text, the parameters (K, h and φ) of the horizontal circular cylinder are estimated and the results are shown in Table 1. It can be noticed that the interpreted results, using the proposed technique, agree well with the assumed values.

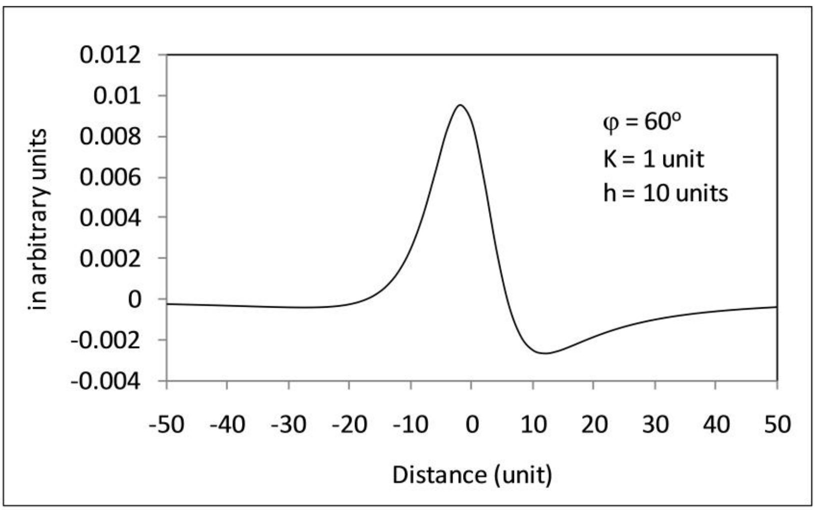

Figure 2. Response of vertical magnetic effect of a 2-D horizontal circular cylinder.

6. Noise analysis

To investigate the noise effect on our estimation method, we added a synthetic anomaly with 5% and 10% of white Gaussian noise (WGN) as shown in Figures 4 and 5. The even components, the odd components, the Hartley transforms and the amplitude spectra of the contaminated anomalies are shown in Figures 6 and 7, respectively. The interpreted results are shown in Table 2. It is clear that the present technique produces satisfactory results even though the anomaly was contaminated with up to 10% of WGN.

Table 1. Synthetic example in arbitrary units

Theoretical Model	K	h	φ
Assumed values	1	10	60^{o}
Interpreted values	1.071	9.94	57.16^{o}
Percentage of error	7.1	6	4.73

Table 2. Synthetic example in arbitrary units, contaminated with 5% and 10% of WGN.

Theoretical Model	*K*	*h*	φ
Assumed values	1	10	60^{o}
Interpreted values with 5% of WGN	0.957	9.686	56.433^{o}
Interpreted values with 10% of WGN	0.912	9.082	57.028^{o}

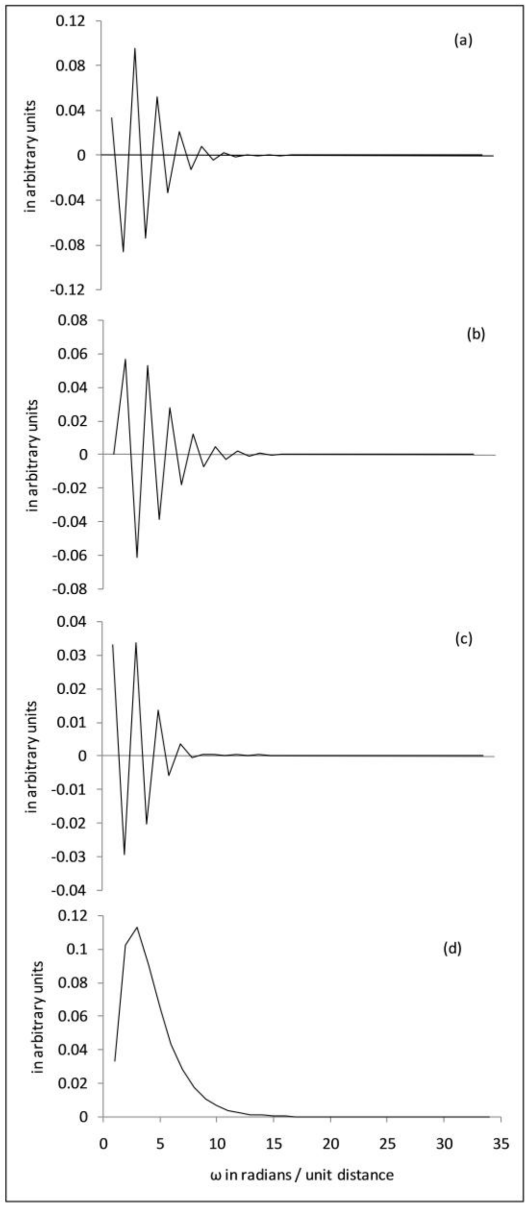

Figure 3. The even component (a), the odd component (b), the Hartley transform (c) and the amplitude spectrum (d) of the horizontal circular cylinder.

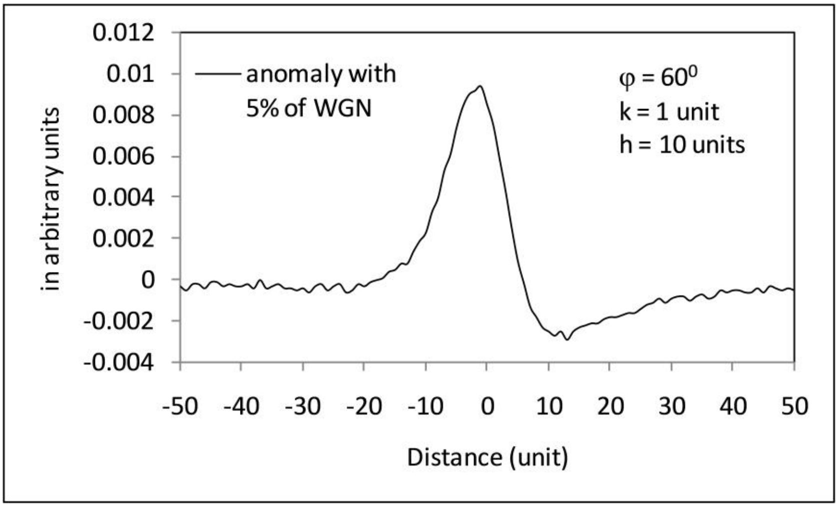

Figure 4. Response of the vertical magnetic effect of a horizontal circular cylinder with 5% of WGN.

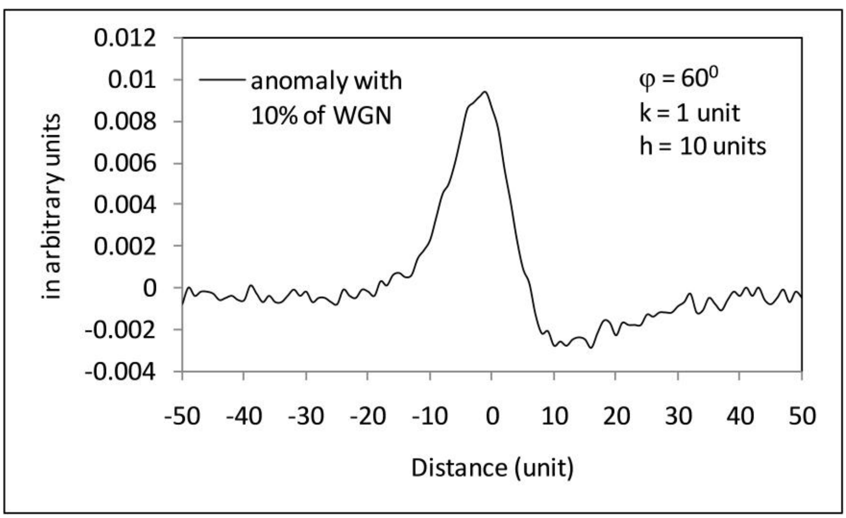

Figure 5. Response of the vertical magnetic effect of a horizontal circular cylinder with 10% of WGN.

7. Field example

The proposed technique is tested with an example of vertical magnetic anomaly over a narrow band of quartz-magnetite in Mangampalli near Karimnagar town, India (Murthy *et al.* 1980), as shown in Figure 8. The anomaly is digitized at 15 ft intervals over 540 ft. The even component, odd component, Hartley transform, and the amplitude spectrum are computed and shown in Figure 9. The interpretation parameters, using the procedures mentioned in the text, are tabulated and shown in Table 3. It shows that the results of the proposed technique inversion are in good agreement with the other published ones.

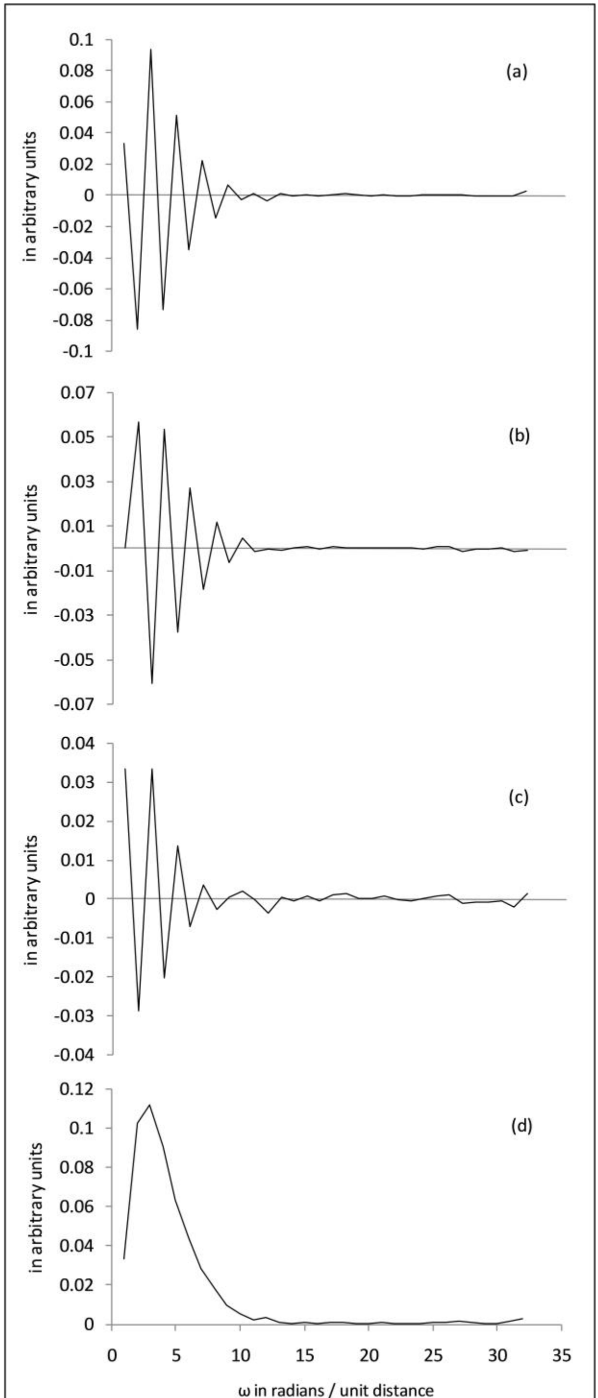

Figure 6. The even component (a), the odd component (b), the Hartley transform (c) and the amplitude spectrum (d) of the horizontal circular cylinder anomaly, contaminated with 5% of WGN.

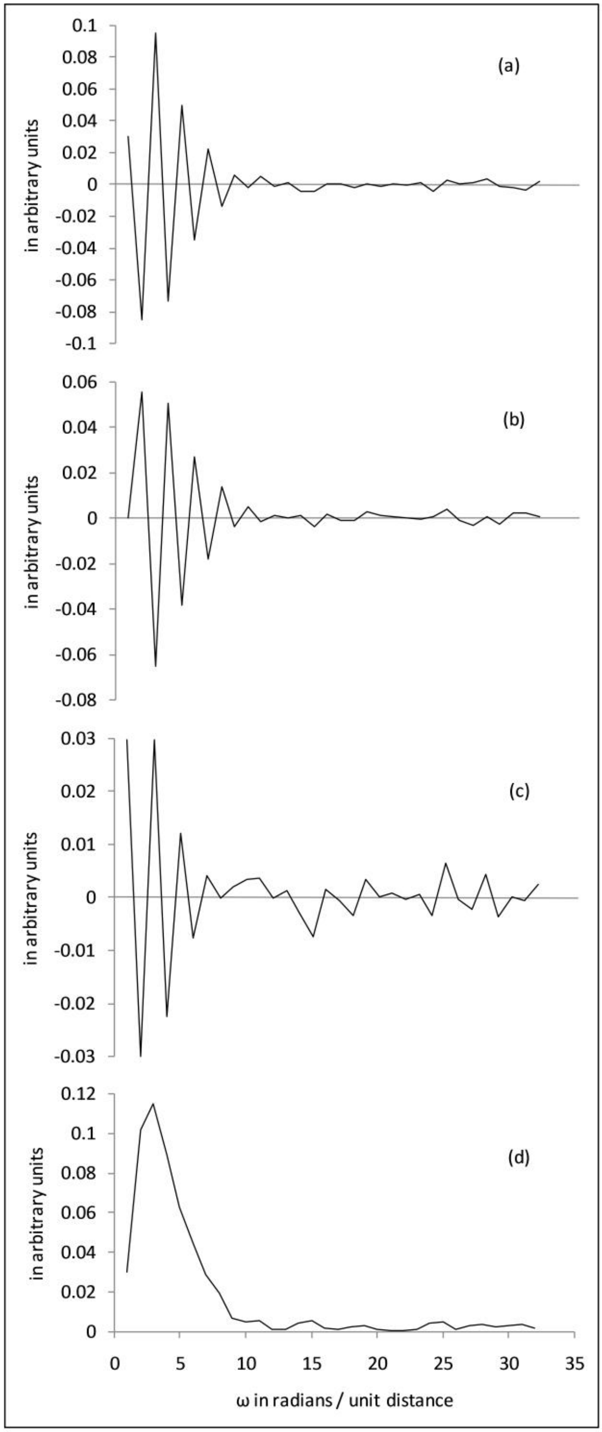

Figure 7. The even component (a), the odd component (b), the Hartley transform (c) and the amplitude spectrum (d) of the horizontal circular cylinder anomaly, contaminated with 10% of WGN.

Table 3. Field example over a narrow band of quartz-magnetite in Mangampalli near Karimnagar town, India

Method	Depth in feet
Murthy *et al*. (1980)	78
Sudhakar *et al*. (2004)	87
Al-Garni (2009)	82.93
Present technique	83.26

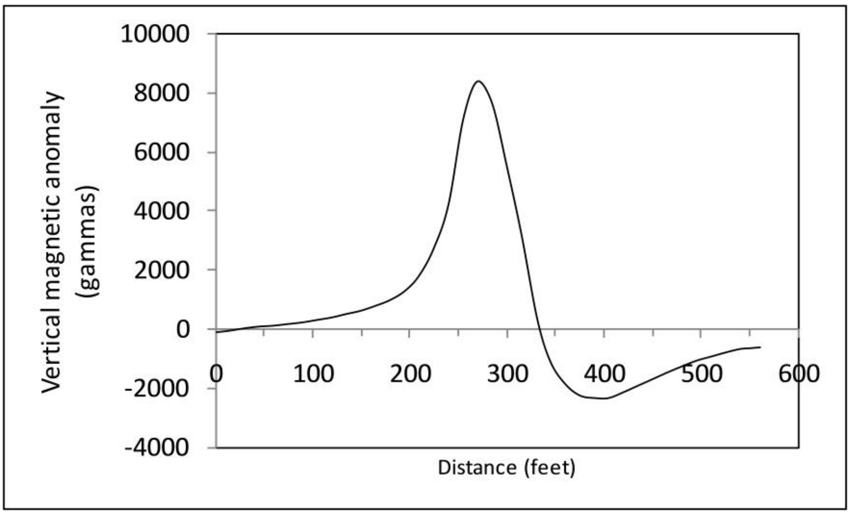

Figure 8. Vertical magnetic anomaly over a narrow band of quartz-magnetite in Mangampalli near Karimnagar town, India (Murthy *et al.* 1980).

8. Conclusion

Spectral analysis, using the Hartley transform, of the magnetic anomalies due to a horizontal circular cylinder has been carried out. This approach was applied first to a synthetic data and then to a real data of the vertical magnetic anomaly over a narrow band of quartz-magnetite in Mangampalli near Karimnagar town. The noises effect on the present technique is tested. This technique shows a level of confidence in the quantitative interpretation of the parameters of the vertical magnetic effect of horizontal cylinder anomalies. Due to the fact that the Hartley transform is purely real, it in general has advantages over the conventional spectral analysis (Fourier transform) in terms of its efficient and economical calculations particularly for more sophisticated problems. It is very interesting to notice that the interpreted results of the real data agree well with those obtained by other techniques, published in the literature.

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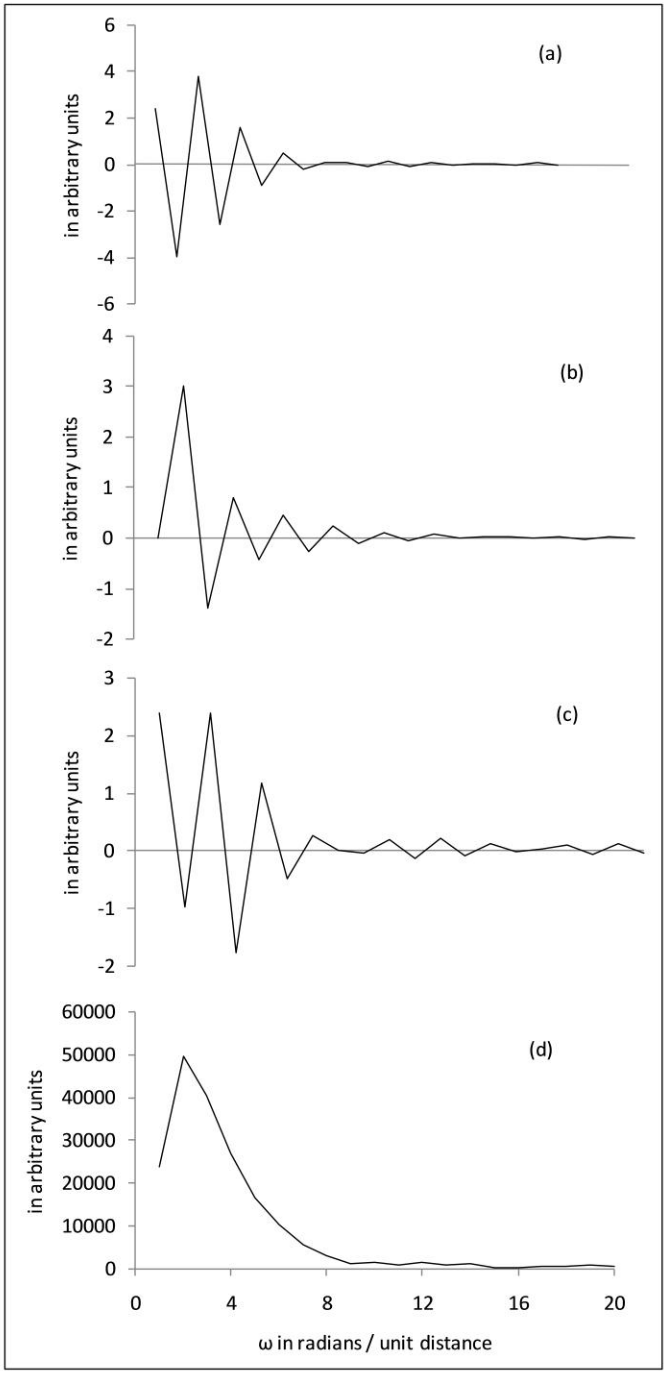

Figure 9. The even component (a), the odd component (b), the Hartley transform (c) and the amplitude spectrum (d) of the vertical magnetic anomaly over a narrow band of quartz-magnetite in Mangampalli near Karimnagar town.

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