

Velocity Profiles for Flow of Omani Crude Oils and Other Liquids

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ABSTRACT: Velocity profiles of Newtonian immiscible liquids undergoing laminar flow between two horizontal plates under pressure gradient are investigated using a momentum balance equation. The differential equation describing the flow has been solved and equations for the velocity profiles of a two-layer and three-layer liquid systems are presented. As examples, we show flow patterns of two-layer water-crude oil system and three-layer system involving water, tetrachloromethane, xylene, cyclopentane and hexane. A distinctive pattern is noticeable between the velocity profiles of heavy (API 19.19) and light (API 40.89) Omani crude oils.

Keywords: Multi-layer flow; Immiscible liquids; Velocity profiles.

أشكال السرعة لتدفق النفط العماني الخام والسوائل الأخرى

سيد العارفين و مجيب الرحمن

ملخص: يستجلى البحث تمثيل السرعة للسوائل النيوتونية غير القابلة للذوبان ذات الاندفاع الصفحي بين سطحين أفقيين تحت تغير في الضغط باستخدام معادلة توازن كمية التحرك. تم حل المعادلات التفاضلية التي تصف التدفق وكذلك معادلات السرعة بين سطحين أو ثلاثة أسطح. كمثال على ذلك، حركة الماء والنفط الخام بين سطحين أو ثلاثة باستخدام الماء و تيتراكلوروميثين و زایلين و سايكلوبنتين والهيكسين. يُمكن أن يلاحظ الشكل المتميز بين السرعات للنفط العماني الخام الخفيف والثقيل (API 19.19 و API 40.89).

مفتاح الكلمات: تدفق متعدد الطبقات، سوائل غير قابلة للاختلاط و شكل تغير السرعة.

1. Introduction

Multi-layer fluid flows span a wide field covering a multitude of various technological perspectives and a broad range of engineering disciplines. Engineering applications of multi-layer flows of immiscible liquids are found in multi-layer extrusion of plastic films, multi-layer coating, lubricated squeezing flows and in the transportation of liquid hydrocarbon [1]. In the two-layer case, like an oil-water system, accurate prediction of flow characteristics, such as flow pattern, water holdup and pressure gradient is important in many engineering applications including petroleum engineering [2]. However, despite their importance, liquid-liquid flows have not been explored to the same extent as gas-liquid flows. In fact, gas-liquid systems represent a very particular extreme of two-fluid systems characterized by low-density and low viscosity ratios. In liquid-liquid systems the density difference between the phases is relatively low while the viscosity ratio encountered in liquid-liquid systems extends over a range of many orders of magnitude. The occurrence of annular flow in liquid-liquid systems is therefore more frequently encountered in oil-water systems of low density differential, and small diameter tubes. Recently Kumara *et al.* [3] used particle image velocimetry (PIV) technique to characterize the flow structure of oil-water flow in horizontal and slightly inclined pipes. They observed stratified flow with some mixing at the interface at mixture velocity of 0.50 m/s for all pipe inclinations.

In addition to these applications in chemical and petroleum engineering, the flow of multi-layered immiscible fluids is also important in the study of the evolution of magma within the Earth. It has been suggested [4] that non-ideal mixing of silicate melts is the cause of liquid immiscibility in simple and naturally occurring systems. Peridotite melts, constituent rock melts of the upper mantle of the Earth, become immiscible as temperatures are raised above their liquidus. Moreover, the kinematic and dynamic behavior of lava flows may dramatically change with lava temperature variations because of the strong temperature-dependence on viscosity [5,6]. Flow during ascent and emplacement of magmas undergoing liquid immiscibility may have important consequences in magma rheology and differentiation. For

example, strong elongation of liquid-liquid structures may have produced the fine lamination in domes of the Piedra Parada caldera in Argentina [7]. It has been observed [8] in the study of multi-component geologically relevant systems that three-fluid immiscibility in the form of coexistence of an aluminosilicate melt with two hyrous fluids is common in natural magmatic systems.

The Earth's outer core is 10% less dense than the pure metallic Fe-Ni liquid [9]. This led geophysicists and geochemists to assume that the outer core may have light elements like sulfur, carbon, oxygen, hydrogen and silicon to account for the 10% density reduction. The same argument is also directed to the study of the interior of other planets like Mars and Mercury. Dasgupta *et al.* [10] investigated the near liquidus phase relations in the Fe-C-S ternary system and concluded that liquid immiscibility could induce stratification of planetary cores that are entirely molten or at least have a molten outer core at pressures less than 6 GPa. Such immiscibility is not expected at the molten outer core of the planets like the Earth, Venus and Mercury where the relevant core pressures are much higher than 6 GPa. This observation contradicts an earlier prediction [11] that there exists immiscible layers of 12 km thickness at the outer liquid core of the Earth. The presence of a thin layer at the top of the outermost core is of considerable importance for the geodynamic and geomagnetic behavior of the Earth; however, detecting the presence of such a thin layer is difficult [12].

Analytical solutions for two-layer flow have been obtained by several authors [13,14]. However, analytical expressions for more than two-layers are not available in the literature because of the 'tedious algebra' involved in deriving the equations for the velocity. Owing to this difficulty it has been suggested [15] that the differential equations can best be solved by standard numerical techniques. In the present study we present exact solutions of the momentum balance equations involving velocity profiles for a three-layer liquid system having different viscosities and densities; this is one-step forward in understanding immiscible multi-layer fluid dynamics. We are, however, presently working towards finding a generalized solution for an n -layer liquid system, which will consider instability of the liquid interfaces.

The layout of the paper is as follows: in section 2, we describe the mathematical formalism of the momentum balance equation and set up the equations for velocity profiles in terms of the unknown constants of integration for an n -layer liquid system. In section 3, we present the solutions of the relevant differential equations for two- and three-layer immiscible liquid systems. Some examples of velocity profiles for two-layer systems involving water and Omani crude oils, and three-layer systems involving water and some chemicals are given in section 4 which is followed by conclusions in section 5.

2. Formalism

The formalism is based on solving differential equations describing momentum balance and Newton's equation for viscosity for an n -layer liquid system as shown in Figure 1. It is assumed that the density of the liquids decreases progressively upwards with the bottom most layer being the most dense and the top the least dense. For steady state flow, the well-known momentum balance equation is [16]

$$\sum_i F_i = \left(\frac{\partial p}{\partial t}\right)_{in} - \left(\frac{\partial p}{\partial t}\right)_{out} \quad (1)$$

where p is the momentum and its time derivatives are the rate of change of momentum entering into and leaving the system respectively. The F 's are the pressure forces acting on the surface and gravity forces acting on the volume as a whole. We assume that (i) the liquid-liquid interface is a plane of constant x coordinate and (ii) tangential components of velocity v_y , v_z and of stress-tensor τ_{xx} , τ_{xy} , and τ_{xz} are continuous throughout an interface (i.e. there is no slip across an interface). The above boundary conditions imply that no material is diffusing through the interface. In other words there is no absorption, adsorption, dissolution, evaporation, melting or chemical reaction at the interface.

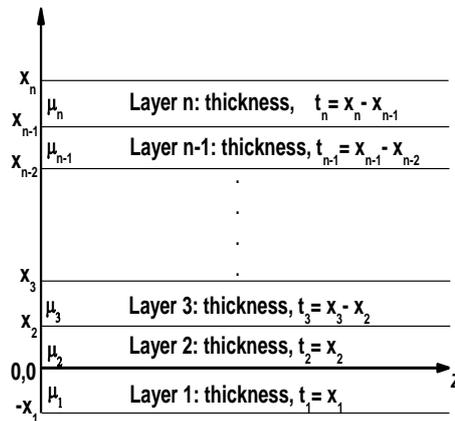


Figure 1. A Schematic diagram for a multi-layer liquid system with varying viscosities and densities.

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The density of the layers is assumed to increase progressively downwards with the bottom most layer being the most dense and the top the least dense.

When two or more immiscible fluids undergoing laminar flow are present in a horizontal thin slit of length L and width W under the influence of a pressure gradient, the velocity distribution can be obtained by solving two governing equations. One of these equations is the momentum balance equation given by:

$$\left(\frac{d\tau_{xz}}{dx}\right) = \frac{\Delta P}{L} \quad (2)$$

and the other one is the Newton's law of viscosity which is given by:

$$\tau_{xz} = -\mu \frac{dv_z}{dx} \quad (3)$$

Here τ_{xz} is the viscous momentum flux component or simply the viscous stress component. τ_{xz} refers to the stress acting in the z direction on the face of a parallelepiped which is perpendicular to the x axis. $\Delta P/L$ is the pressure gradient, where ΔP is the pressure difference over a length L of the fluid surface, μ is the viscosity, and v_z is the velocity. Integration of (2) yields

$$\tau_{xz} = \left(\frac{\Delta P}{L}\right)x + C \quad (4)$$

where C is a constant of integration. Using the assumption of continuous τ_{xz} throughout any of the interfaces, it can be shown that C is the same for all the interfaces within a given number of immiscible liquids. C changes if the number of given liquids changes and henceforth it will be denoted by ${}^n C$. The index n is the number of liquids present in a system and its minimum value is 2, indicating the minimum number of layers required to constitute a liquid system in this study. Substituting (3) in (4) and integrating, one can write the equation for velocity for an n -layer liquid system as:

$${}^n v_{iz} = -\left(\frac{\Delta P}{2\mu_i L}\right)x^2 - \frac{{}^n C}{\mu_i}x + {}^n d_i \quad (5)$$

where μ_i and ${}^n v_{iz}$ are respectively the viscosity and velocity of liquid in the i^{th} layer of an n -layer liquid system. The running index i takes on values 1, 2, 3, ..., n . The $(n+1)$ constants of integration, ${}^n C$ and ${}^n d_i$, are to be determined by applying the following three boundary conditions in (5)

(i) at $x = -x_1$, i.e. at the bottom solid surface, ${}^n v_{1z} = 0$

(ii) at $x = 0$, i.e. at the interface between layer 1 and layer 2 (see figure 1), ${}^n v_{1z} = {}^n v_{2z}$ and

(iii) at $x = x_k$, i.e. for all interfaces between the 2^{nd} layer and the $(n-1)^{\text{th}}$ layer, ${}^n v_{kz} = {}^n v_{(k+1)z}$ where $k = 2, 3, 4, \dots, n-1$ and at $x = x_n$, i.e. at the top solid surface, ${}^n v_{nz} = 0$

and solving the resulting $(n+1)$ equations. Applying boundary condition (i) in Eq. (5) one gets

$$\left(-\frac{\Delta P}{2L\mu_1}\right)x_1^2 + \frac{{}^n C}{\mu_1}x_1 + {}^n d_1 = 0 \quad (6)$$

The boundary condition (ii) at $x = 0$ gives:

$${}^n d_1 = {}^n d_2 \quad (7)$$

while the boundary condition (iii) at $x = x_k$ (i.e. the interface between k^{th} and $(k+1)^{\text{th}}$ layers) gives

$$\left(-\frac{\Delta P}{2L\mu_k}\right)x_k^2 - \frac{{}^n C}{\mu_k}x_k + {}^n d_k = \left(-\frac{\Delta P}{2L\mu_{k+1}}\right)x_k^2 - \frac{{}^n C}{\mu_{k+1}}x_k + {}^n d_{k+1} \quad (8)$$

Likewise the boundary condition (iii) at $x = x_n$, ${}^n v_z = 0$ yields

$${}^n v_z = \left(-\frac{\Delta P}{2\mu_n L}\right)x_n^2 - \frac{{}^n C}{\mu_n}x_n + {}^n d_n = 0 \quad (9)$$

The task is now to solve Eqs. (6), (7), (8) and (9) to determine the $(n+1)$ constants of integration such as C arising out of Eq. (4) and the ${}^n d_i$ ($i = 1, 2, 3, \dots, n$) arising out of Eq. (5). We will restrict our investigations in this study to two- and three- layer liquid systems only. Our investigation for a generalized solution for an n -layer system is in progress.

3. Solutions of the momentum balance equations

We present here solutions of the momentum balance equations involving two- and three- layer liquid systems only. The constants (i.e. the C 's in Eq. 4 and the d 's in Eq. 5) of integration are first determined by solving Eqs. (6), (7), (8) and (9) and then substituting back in Eq. (5) to find the velocities within each layer. These velocities follow a pattern that can be represented by a mathematical series. We represent the velocity by ${}^n v_i$ after dropping the subscript z . It is understood that all velocities considered in the present study are directed along the z -axis (see Figure 1). The superscript, n indicates the total number of layers in a system and its minimum value is 2. The subscript, i ($i = 1, 2, 3, \dots, n$) represents an arbitrary layer for which the d 's and v 's are needed.

3.1 A two-layer liquid system

For a two-layer liquid system $n = 2$. We will consider two simultaneous equations, Eqs. 6 and 8 to find the constant 2C and either 2d_1 or 2d_2 (because ${}^2d_1 = {}^2d_2$). We present the constants and the velocities in terms of the thicknesses (Figure 1) of the layers so that

$${}^2C = A \left[\frac{t_1^2 \mu_2 - t_2^2 \mu_1}{t_1 \mu_2 + t_2 \mu_1} \right] \quad (10a)$$

$${}^2d_1 = {}^2d_2 = \frac{At_2 \{t_1(t_1 + t_2)\}}{t_1 \mu_2 + t_2 \mu_1} \quad (10b)$$

The superscript 2 represents the total number of layers in the liquid-liquid system and the subscripts represent an arbitrary layer within the system. We define numerator, Num and denominator, $Deno$ for the 2-layer system as

$${}^2Num = t_1^2 \mu_2 - t_2^2 \mu_1 \quad (10c)$$

and

$${}^2Deno = t_1 \mu_2 + t_2 \mu_1 \quad (10d)$$

so that the constant related to the viscous stress component can be denoted by 2C by the equation

$${}^2C = A \frac{{}^2Num}{{}^2Deno} \quad (10e)$$

where $A = \frac{\Delta p}{2L}$. The velocities in the two layers are

$${}^2v_1 = \frac{A}{\mu_1} (t_1 + x) \left[(t_1 - x) - \frac{{}^2C}{A} \right] \quad (10f)$$

and

$${}^2v_2 = \frac{A}{\mu_2} (t_2 - x) \left[(t_2 + x) - \frac{{}^2C}{A} \right] \quad (10g)$$

3.2 A three-layer liquid system

In the case of a three-layer system, $n = 3$. The numerator, Num and denominator, $Deno$ for three layers are

$${}^3Num = A \left[\begin{array}{l} t_1^2 \mu_2 \mu_3 - t_2^2 \mu_3 \mu_1 \\ -t_3(2t_2 + t_3) \mu_1 \mu_2 \end{array} \right] \quad (11a)$$

and

$${}^3Deno = \left(\begin{array}{l} t_1 \mu_2 \mu_3 + t_2 \mu_3 \mu_1 \\ + t_3 \mu_1 \mu_2 \end{array} \right) \quad (11b)$$

The constants of integrations for a three-layer system are given by:

$${}^3C = A \frac{{}^3Num}{{}^3Deno} \quad (11c)$$

$${}^3d_1 = {}^3d_2 = \frac{[A(t_2 + t_3)\{t_1(t_1 + t_2 + t_3)\mu_2\} + At_2\{t_1(t_1 + t_2)(\mu_3 - \mu_2)\}]}{{}^3Deno} \quad (11d)$$

and

$${}^3d_3 = \frac{[A(t_2 + t_3)\{t_1(t_1 + t_2 + t_3)\mu_2\mu_3\} + t_2 t_3 \mu_1 (\mu_3 - \mu_2)]}{\mu_3 {}^3Deno} \quad (11e)$$

It may be recalled that the constant of integration, C arising out of momentum balance equation (Eq. 2) is the same for all interfaces of an n -layer system because viscous stress component, τ_{xz} is assumed to be continuous throughout the interfaces.

The velocities for the three-layer system are given by

$${}^3v_1 = \frac{A(t_1 + x)}{\mu_1} \left[(t_1 - x) - \frac{{}^3C}{A} \right] \quad (11f)$$

and

$${}^3v_2 = \frac{A(t_2 - x)}{\mu_2} \left[(t_2 + x) + \frac{{}^3C}{A} \right] + {}^3AVT_2 \quad (11g)$$

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where 3AVT_2 is the additional velocity term added to the velocity in the second layer of the 3-layer liquid system. It is given by:

$${}^3AVT_2 = \frac{At_3\{t_1(t_1+2t_2+t_3)\mu_2+t_2(t_2+t_3)\mu_1\}}{{}^3Deno} \quad (11h)$$

The velocity in the third layer is given by:

$${}^3v_3 = \frac{A(t_3-x)}{\mu_3} \left[(t_3 + x) + \frac{{}^3C}{A} \right] \quad (11i)$$

The superscript 3 indicates the number of liquids in the system and the subscripts 1, 2 and 3 represent the individual layers in which the velocities are to be found.

4. Results and discussions

We have used MATLAB to calculate the velocity profiles for two- and three-layer immiscible liquid systems. In all these cases we have assumed that the density of the liquids decrease upwards so that the most dense liquid is in the bottom most layer and the least dense liquid is in the top most layer. Our entire analysis is based on stratified flow regime. We have used water, Omani crude oils (Erad and Zal-41) and a number of chemicals such as tetrachloromethane, xylene, dichloromethane, cyclopentane and hexane to perform the model calculations of the velocity profiles in the liquids. The viscosities and densities of these liquids are given in Table 1. The net pressure, ΔP between the ends of the horizontal plates is taken to be 5 Pa for all the cases.

Table 1. Densities and viscosities of liquids used in computing the velocity profiles for two- and three-layer systems.

Properties Liquids	Density ($\text{g}\cdot\text{cm}^{-3}$)	Viscosity ($\text{mPa}\cdot\text{s}$)
Tetrachloromethane	1.595	1.460
Water	1.000	1.002
Erad crude	0.933	580
Xylene	0.880	0.812
Zal-41	0.818	9.900
Cyclopentane	0.751	0.440
Hexane	0.655	0.294

4.1 Omani crude oils

We have chosen two representative samples of Omani crude oils to study the flow dynamics in water-crude oil system. Erad is described as a heavy crude oil (API 19.19) and has the largest viscosity (0.58 Pa s) while Zal – 41 is described as a light crude oil (API 40.89) with the smallest (0.0093 Pa s) among the samples we have investigated [17,18]. The results for the simplest two-layer system consisting of water and Omani crude oils are shown in Figures. 2a, 2b, 2c and 2d for various crude oil and water thicknesses treated within stratified flow approximation. In the case of the Erad-water system, the water thickness is kept at 1.0 mm (Figures 2a and 2b) while the oil thickness is doubled (Figure 2b). By doubling the oil thickness in Figure 2b, the maximum velocity of the water layer has increased by about 29% but the velocity at the interface has increased by about 233%. The net pressure Δp is kept at 5 Pa in both the cases. Figures 2c and 2d show the velocity distributions for Zal-41 crude oil and water system. It may be noted that the Zal – 41 sample has a very low viscosity (0.0093 Pa s) in comparison with that of the Erad sample (0.58 Pa s). In this case the oil thickness was kept constant at 5 mm while the water thickness is different, 1 mm in Figure 2c and 3 mm in Figure 2d. An increase of water thickness by 300% has increased the maximum velocity by a factor of 10 for the same net pressure of 5 Pa.

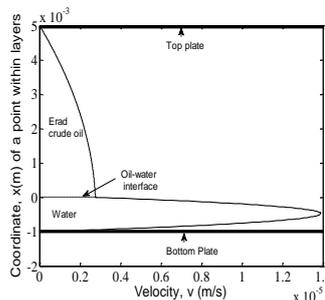


Figure 2a. Velocity profile of Erad (heaviest) crude oil-water system with an oil thickness 5 mm.

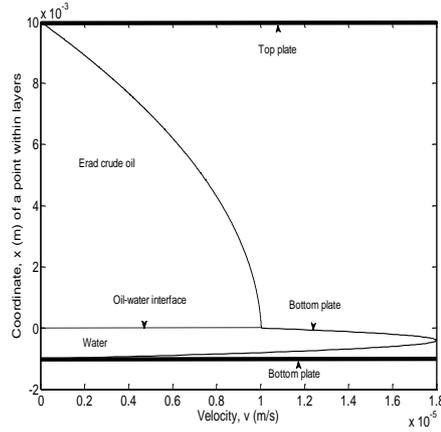


Figure 2b. Velocity profile of Erad (heaviest) crude oil-water system with an oil thickness 10 mm.

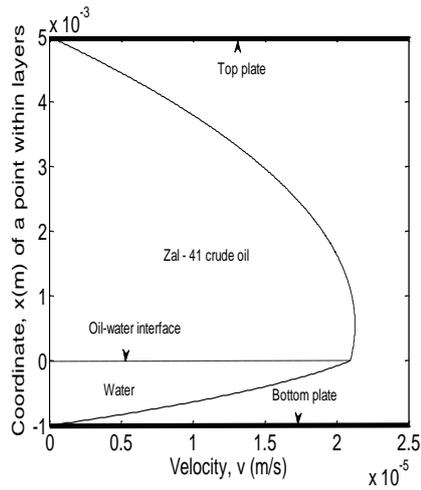


Figure 2c. Velocity profile of Zal-41 (lightest) crude oil-water system with a water thickness of 1mm and an oil thickness 5 mm.

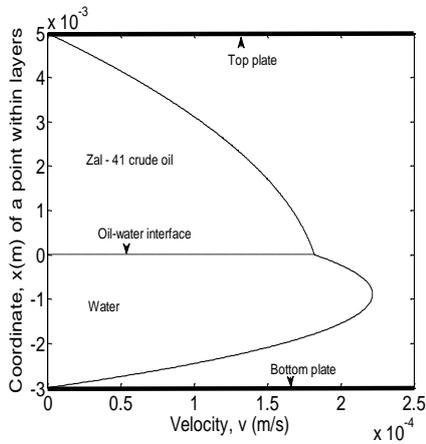


Figure 2d. Velocity profile of Zal-4a (lightest) crude oil-water system with a water thickness of 3 mm and an oil thickness of 5 mm.

4.2 Other liquids

The effects of the viscosity of the top layer in a 3-layer system consisting of the liquids tetrachloromethane, water, xylene, cyclopentane and hexane are shown in Figures 3 (a-c). Thicknesses of the layers and the pressure difference are kept same in all these three cases.

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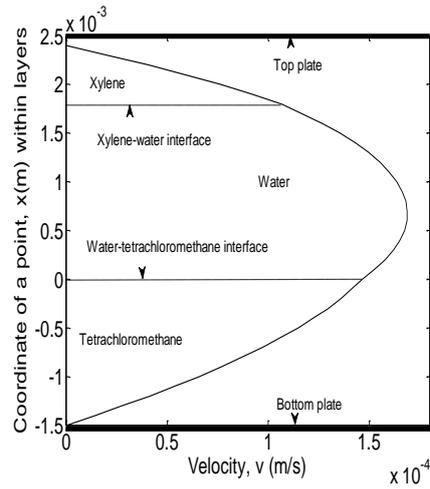


Figure 3a. Velocity profile of tetrachloromethane-water-xylene system.

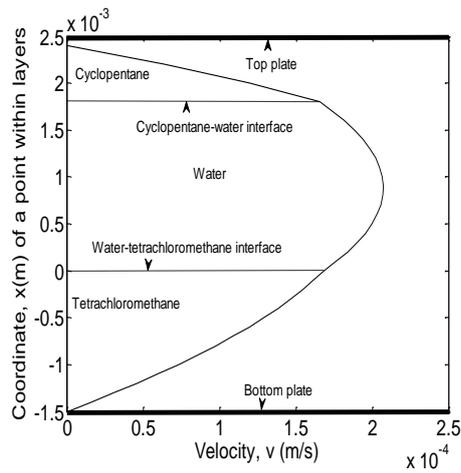


Figure 3b. Velocity profile of a tetrachloromethane-water-cyclopentane system.

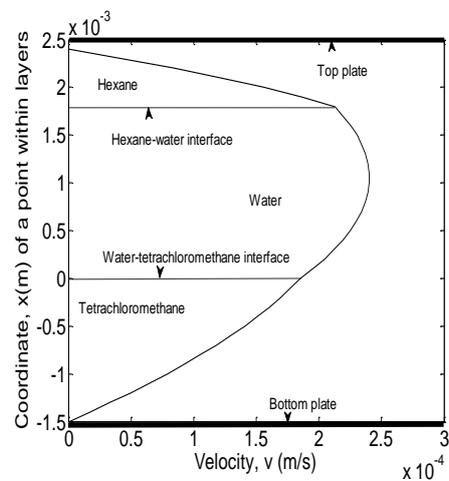


Figure 3c. Velocity profile of a tetrachloromethane-water-hexane system.

The maximum velocity increases as the the viscosity of the topmost layer decreases. The maximum velocities are respectively 0.170 , 0.207 and 0.241 mm s^{-1} for xylene (Figure 3a), cyclopentane (Figure 3b) and hexane (Figure 3c) as the topmost layer. The interface velocities also follow the same trend as that of the maximum velocity.

5. Conclusions

The differential equations involving multi-layer flow of liquids under ideal conditions of Newtonian liquids satisfying laminar and stratified flow have been set up for an n -layer liquid system and solutions for the velocity profiles of two- and three-layer systems are presented. These solutions are exact analytical expressions for the velocity profiles within the liquids. As examples, flow patterns of two-layer water-Omani crude oil systems and three-layer systems involving tetrachloromethane, xylene, cyclopentane and hexane have been determined. The flow pattern for the heaviest Erad crude oil is distinctively different from that of the lightest Zal-41 crude oil.

The viscosity data used in the calculation of the velocity profile are presented in Table 1. For almost all of the model fluids the viscosity is lower than that of water at room temperature being the viscosity of the first layer *i.e.* tetrachloromethane (1.460 mPa s).

It is relevant to mention here that in any real multi-layer system, the presence of absorption, adsorption dissolution, evaporation, melting or chemical reactions will complicate the situation. Most of these processes are directly related to wetting and friction phenomena complicating the flow dynamics. In some circumstances interfacial instability occurs causing turbulence in the flow process. This real picture will certainly require modification of our approximations and in our future endeavor we will address the above issues, incorporating an extensive momentum balance equation along with exact solutions for n -layer liquid system.

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