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Travelling Wave Solutions for Fisher's Equation Using the Extended Homogeneous Balance Method

Mohammad M. Fares[1], Usama M. Abdelsalam[1,2]* and Faiza M. Allehiany[3]

[1]Department of Mathematics, University of Technology and Applied Sciences - Al Rustaq, Oman; [2]Department of Mathematics, Faculty of Science, Fayoum University, Egypt; [3]Department of Mathematics, Faculty of Applied Sciences, Umm Al-Qura University, Mecca, Saudi Arabia. *Email: usamaahmad.rus@cas.edu.om.

ABSTRACT: In this work, the extended homogeneous balance method is used to derive exact solutions of nonlinear evolution equations. With the aid of symbolic computation, many new exact travelling wave solutions have been obtained for Fisher's equation and Burgers-Fisher equation. Fisher's equation has been widely used in studying the population for various systems, especially in biology, while Burgers-Fisher equation has many physical applications such as in gas dynamics and fluid mechanics. The method used can be applied to obtain multiple travelling wave solutions for nonlinear partial differential equations.

Keywords: Traveling wave solutions; Partial differential equations; Extended homogeneous balance method and Fisher's equation.

الحلول الموجية لمعادلة فيشر باستخدام طريقة التوازن المتجانس الموسعة

محمد محمد فارس، أسامة محمد عبدالسلام و فايزة محمد اللحياني

الملخص: في هذا العمل ، تم استخدام طريقة التوازن المتجانس الموسعة لاشتقاق الحلول الدقيقة (التحليلية) لبعض المعادلات التفاضلية الجزئية. بمساعدة بعض البرامج الحسابية ، تم الحصول على العديد من الحلول الموجية الدقيقة الجديدة لمعادلة فيشر ومعادلة برجر فيشر. تم استخدام معادلة فيشر على نطاق واسع في دراسة السكان لأنظمة مختلفة ، وكذلك في علم الأحياء ، كما أن معادلة برجر فيشر لها العديد من التطبيقات الفيزيائية مثل ديناميكيات الغاز وميكانيكا الموائع. يمكن تطبيق الطريقة المستخدمة للحصول على العديد من الحلول الموجية للمعادلات التفاضلية الجزئية غير الخطية

الكلمات المفتاحية: الحلول الموجية ، المعادلات التفاضلية الجزئية ، طريقة التوازن المتجانس الموسعة ، معادلة فيشر.

1. Introduction

The exact travelling wave solutions of nonlinear evolution equations play an important role in the study of nonlinear physical phenomena, for example, the wave phenomena observed in fluid mechanics, plasma physics, optical fibers, solid state physics, chemical kinematics, chemical physics and geochemistry. Explicit solutions to the mathematical modelling of physical problems are of fundamental importance. Many methods have been developed for finding the exact solutions of nonlinear evolution equations, such as the inverse scattering method [1, 2], bilinear transformation [1, 3, 4], the tanh-function method [5, 6], extended tanh method [7-10], sine-cosine method [11, 12], F-expansion method [13], general expansion method [14, 15], and (G'/G) method [16-18]. The homogeneous balance (HB) method, which is a direct and effective algebraic method for the computation of exact traveling wave solutions, was first proposed by Wang [19, 20]. Later [21,22], the HB method was extended to search for other kinds of exact solutions. Fan [23] used the HB method to search for Backlund transformation and similarity reduction of nonlinear PDEs. He also showed that there is a close connection among the HB method, Weiss, Tabor, Carnevale (WTC) method and Clarkson, Kruskal (CK) method. The extended homogeneous balance method is used to solve many nonlinear evolution equations [24-28].

The Fisher's equation [29,30] is a nonlinear partial differential equation of second order.

$$u_t = u_{xx} + u(1-u).$$

Fisher proposed this equation as a model for the propagation of a mutant gene with $u(x,t)$ denoting the density of advantages. This equation is encountered in chemical kinetics, population dynamics, flame propagation, autocatalytic chemical reactions and branching Brownian motion processes. The aim of this work is to propose an extension of the homogeneous balance method to construct more other kinds of exact solutions to nonlinear PDEs. In order to illustrate the effectiveness and convenience of the method, the method is applied to Fisher's equation and Burgers-Fisher equation.

In the following section, let us simply describe the extended homogeneous balance method.

2. Proposed analytical method

In general, consider a given PDE, say in two variables

$$H(u, u_t, u_x, u_{xx}, \ldots.) = 0. \tag{1}$$

We seek for the special solution of Eq. (1), the travelling wave solution, in the form

$$u(x,t) = u(\zeta), \qquad \zeta = x - \lambda\, t, \tag{2}$$

where ϑ and L are constants to be determined later. Using the transformation (2), Eq. (1) reduces to a nonlinear ordinary differential equation (ODE). The next crucial step is that the solution we are looking for is expressed in the form

$$u(\zeta) = \sum_{i=0}^{n} a_i \omega^i + \sum_{i=1}^{n} b_i [1+\omega]^{-i}, \tag{3}$$

and

$$\omega' = k + M\omega + P\omega^2, \tag{4}$$

where a_i and b_i are constants, while k, M and P are parameters to be determined later, $\omega = \omega(\zeta)$, and $\omega' = d\omega/d\zeta$. The mechanism for solitary wave solutions to occur is the fact that different effects (such as, the dispersion and nonlinearity) that act to change the wave forms in many nonlinear physical equations have to balance each other. Therefore, one may use the above fact to determine the parameter n, which must be a positive integer, and can be found by balancing the highest-order linear term with the nonlinear terms [26]. Substituting (3) and (4) in the relevant ODE will yield a system of ODEs with respect to a_0, a_i, b_i, k, M, P and λ (where $i = 1, \ldots, m$), because all the coefficients of ω^j (where $j = 0,1, \ldots$) have to vanish. With the aid of *MATHEMATICA*, one can determine a_0, a_i, b_i, k, M, P and λ.

It is to be noted that the Riccati equation (4) can be solved using the homogeneous balance method as follows:

Case I: when P = 1, M = 0, the Riccati Eq. (4) has the following solutions

$$\omega = \{[c]ll - \sqrt{-k}\tanh[\sqrt{-k}\zeta], \;\; with k < 0, -\sqrt{-k}\coth[\sqrt{-k}\zeta], with < 0, \tag{5}$$

$$\omega = -\frac{1}{\zeta}, \qquad \text{with} \;\; k = 0, \tag{6}$$

and

$$\omega = \{[c]ll\sqrt{k}\tan[\sqrt{k}\zeta], \quad with\ k > 0, -\sqrt{k}\cot[\sqrt{k}\zeta], \qquad with\ \ k > 0. \tag{7}$$

Since coth- and cot-type solutions appear in pairs with tanh- and tan-type solutions, respectively, they are omitted in this paper.

Case II:, Let $\omega = \sum_{i=0}^{m} A_i \tanh^i(p_1\zeta)$. Balancing ω' with ω^2 leads to

$$\omega = A_0 + A_1 \tanh(p_1\zeta). \tag{8}$$

Substituting equation (8) into (4), we have the following solution of Eq. (4)

$$\omega = -\frac{p_1}{2P}\tanh(\frac{p_1}{2}\zeta) - \frac{M}{2P}, \text{with } \ Pk = \frac{M^2 - p_1^2}{4}. \tag{9}$$

Similarly, let $\omega = \sum_{i=0}^{m} A_i \coth^i(p_1\zeta)$, then we obtain the following solution:

$$\omega = -\frac{p_1}{2P}\coth(\frac{p_1}{2}\zeta) - \frac{M}{2P}$$

with $Pk = \frac{M^2 - p_1^2}{4}$.

Case III:, We suppose that the Riccati Eq. (4) has the following solutions of the form

$$\omega = A_0 + \sum_{i=0}^{m} (A_i f^i + B_i f^{i-1} g), \tag{10}$$

with

$$f = \frac{1}{\cosh\zeta + r}, \quad g = \frac{\sinh\zeta}{\cosh\zeta + r}, \tag{11}$$

Substituting equations (10) and (11) into (4), we have the following solution of Eq. (4)

$$\omega = -\frac{1}{2P}\left(M + \frac{\sinh(\zeta) + \sqrt{r^2 - 1}}{\cosh(\zeta) + r}\right), \text{with } \ Pk = \frac{M^2 - 1}{4} \tag{12}$$

where r is an arbitrary constant. It should be noticed that solution (12), as $r = 1$, degenerates to

$$\omega = -\frac{1}{2P}\left[M + \tanh\left(\frac{\zeta}{2}\right)\right] \tag{13}$$

Case IV:, We suppose that the Riccati Eq. (4) has the following solutions of the form

$$\omega = A_0 + \sum_{i=0}^{m} \sinh^{i-1}(A_i \sinh\eta + B_i \cosh\eta), \tag{14}$$

where $d\eta/d\zeta = \sinh\eta$ or $d\eta/d\zeta = \cosh\eta$ Balancing ω' with ω^2 leads to $m = 1$

$$\omega = A_0 + A_1 \sinh\eta + B_1 \cosh\eta. \tag{15}$$

When $d\eta/d\zeta = \sinh\eta$, we substitute (15) and $d\eta/d\zeta = \sinh\eta$ into (4) and set the coefficient of $\sinh^i\eta\cosh^j\eta, i = 0,1,2, j = 0,1$ to zero and solve the obtained set of algebraic equations to get

$$A_0 = \frac{-M}{2P}, A_1 = 0, B_1 = \frac{1}{P}, \tag{16}$$

where $k = \frac{M^2 - 4}{4P}$, while

$$A_0 = \frac{-M}{2P}, A_1 = \pm\sqrt{\frac{1}{2P}}, B_1 = \frac{1}{P}, \tag{17}$$

where $k = \frac{M^2 - 1}{4P}$. To $d\eta/d\zeta = \sinh\eta$ we have

$$\sinh\eta = -csesh\zeta, \cosh\eta = -\coth\zeta. \tag{18}$$

From (16)-(18) we obtain

$$\omega = -\frac{M + 2\coth\zeta}{2P}. \tag{19}$$

where $k = \frac{M^2-4}{4P}$, and

$$\omega = -\frac{M \pm csesh\zeta + \coth\zeta}{2P}. \tag{20}$$

where $k = \frac{M^2-1}{4P}$

3. Applications of the proposed method

In this section, we will illustrate the above approach for a class of nonlinear evolution equations namely, **Fisher's equation**.

3.1 Example 1. Fisher's equation

We apply the extended homogeneous balance method to construct the traveling wave solutions for Fisher's equation [24,25]. The Fisher's equation is a nonlinear partial differential equation of second order, of the form

$$u_t = u_{xx} + u(1-u). \tag{21}$$

Applying the transformation $u(x,t) = U(\zeta), \zeta = x - \lambda t$ to Eq. (21) we find V satisfies the following ordinary differential equation

$$-U + U^2 - \lambda U' - U'' = 0. \tag{22}$$

Balancing U'' with U^2 yields $|m| = 2$.Therefore, we are looking for the solution in the form

$$U = a_0 + b_0 + a_1\omega + b_1(1+\omega)^{-1} + a_2\omega^2 + b_2(1+\omega)^{-2}, \tag{23}$$

Substituting Eqs. (23) and (4) in Eq. (22), we get a polynomial equation ω. Hence, equating the coefficient of ω^j (i.e., $j = 0,1,2,\ldots$) to zero and solving the obtained system of overdetermined algebraic equation using symbolic manipulation package MATHEMATICA, results in:

The first set:

$$a_1 = 0, P \neq 0, a_0 = \frac{1}{2}(-12P^2 + 12kP + 1), b_1 = \frac{1}{2}\sqrt{3}\sqrt{-384P^4 + 576kP^3 - 192k^3P + 1}, a_2 = 0,$$
$$b_2 = 6(k^2 - 2Pk + P^2), \lambda = -480(kP^2 b_1 - P^3 b_1). \tag{24}$$

The second set:

$$b_2 = 0, P \neq 0, k = \frac{M^2-1}{4P}, a_0 = \frac{1}{2}(M^2 + 8kP + 1), b_1 = 0, a_2 = 6P^2, M \neq 0,$$

$$= \frac{-36PM^6 + 432kP^2M^4 - 1728k^2P^3M^2 + 211PM^2 + 2304k^3P^4 - 424kP^2 - 35P + 70Pa_0}{35M},$$
$$\lambda = \frac{6}{7}(36M^9 - 432kPM^7 + 1728k^2P^2M^5 - 73M^5 - 2304k^3P^3M^3 + 160kPM^3 + 70a_0M^3 - 35M^3$$
$$-1152k^2P^2M + 280kPM - 560kPa_0M + 2M + 560k^2Pa_1). \tag{25}$$

For the first set (24), if $M = 0$, $P = 1$ we get the solutions satisfying case I for $k > 0$. Therefore, the solutions of Fisher's equation of the type (21), will be

$$u_1(x,t) = a_0 + \frac{b_2 + b_1(\sqrt{k}\tan(\sqrt{k}\zeta) + 1)}{(\sqrt{k}\tan(\sqrt{k}\zeta)+1)^2}, \tag{26}$$

$$u_2(x,t) = a_0 + \frac{(\sqrt{k}\cot(\sqrt{k}\zeta)+1)b_1 + b_2}{(\sqrt{k}\cot(\sqrt{k}\zeta)+1)^2}. \tag{27}$$

For $k < 0$,

$$u_3(x,t) = a_0 + \frac{b_2 + b_1\left(1 - \sqrt{-k}\tanh(\sqrt{-k}\zeta)\right)}{(\sqrt{-k}\tanh(\sqrt{-k}\zeta)-1)^2}, \tag{28}$$

$$u_4(x,t) = a_0 + \frac{\left(1-\sqrt{k}\coth(\sqrt{k}\zeta)\right)b_1 + b_2}{\left(\sqrt{k}\coth(\sqrt{k}\zeta)-1\right)^2}, \tag{29}$$

Now for the solutions satisfying cases II & III & IV, we have the compatibility condition,

$$Pk = \frac{M^2 - p_1^2}{4}. \tag{30}$$

Therefore, substitute for P and k, from Eq. (24) into Eq. (30) and solve for p_1. It is found that

$$p_1 = -\frac{\sqrt{1-2a_0}}{\sqrt{3}} \quad \text{or} \quad \frac{\sqrt{1-2a_0}}{\sqrt{3}}. \tag{31}$$

Hence, for case II, we get the following solutions:

$$u_5(x,t) = a_0 + \frac{2P\Big(2Pb_2 + b_1\big(2P - p_1(M + 2\tanh(\zeta p_1))\big)\Big)}{\left(p_1(M+2\tanh(\zeta p_1)) - 2P\right)^2}, \tag{32}$$

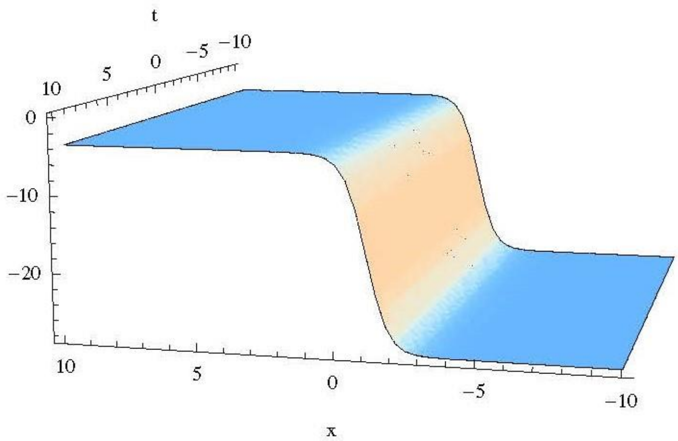

Figure 1. 3D and contour plots of the solution (32) with $a_0 = -2$ $P = 5$ and $k = 5$.

and

$$u_6(x,t) = a_0 + \frac{2P\left(2P(b_1+b_2) - (M + 2\coth(\zeta p_1))b_1 p_1\right)}{\left((M+2\coth(\zeta p_1))p_1 - 2P\right)^2}, \tag{33}$$

In the same manner case III, results in the solution

$$u_7(x,t) = a_0 + \frac{4P^2 b_2 (r+\cosh(\zeta))^2}{\left(Mr - 2Pr + (M-2P)\cosh(\zeta) + \sinh(\zeta) + \sqrt{r^2-1}\right)^2} - \frac{2Pb_1(r+\cosh(\zeta))}{Mr - 2Pr + (M-2P)\cosh(\zeta) + \sinh(\zeta) + \sqrt{r^2-1}}, \tag{34}$$

with the condition that $p_1 = 1$.

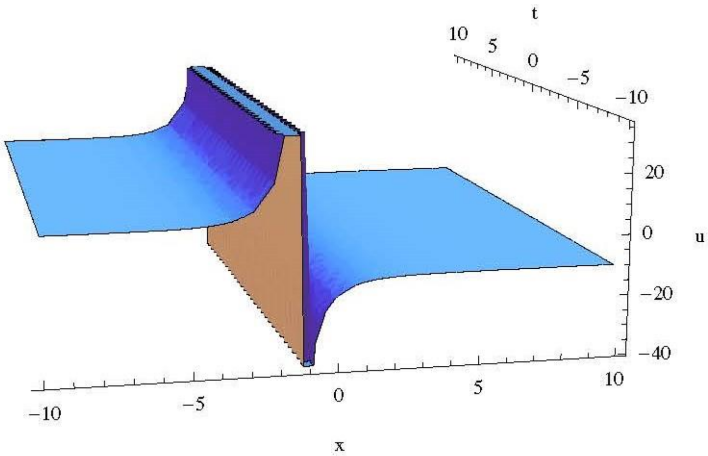

Figure 2. 3D and density plots of the solution (35) with $a_0 = -2$, $P = 5$ and $k = 5$.

For case IV, the solution form is

$$u_8(x,t) = a_0 + \frac{2P((M+2P+\coth(\zeta)+\operatorname{csch}(\zeta))b_1+2Pb_2)}{(M+2P+\coth(\zeta)+\operatorname{csch}(\zeta))^2}, \tag{35}$$

with the condition that $p_1 = 1$,

$$u_9(x,t) = a_0 + \frac{2P((-M+2P-2\coth(\zeta))b_1+2Pb_2)}{(M-2P+2\coth(\zeta))^2}, \tag{36}$$

with the condition that $p_1 = 2$.

For the second set we are left only with solutions satisfying cases II & III & IV. Since, the main criteria for these cases to be applicable is the compatibility condition,

$$Pk = \frac{M^2-p_1^2}{4}. \tag{37}$$

From (25), it is found that

$$p_1 = 1. \tag{38}$$

Therefore, solutions to equation of the type (21), will be

$$u_{10}(x,t) = a_0 + \frac{3}{2}p_1^2\left(M + 2\tanh(\zeta p_1)\right)^2 - \frac{a_1p_1\left(M+2\tanh(\zeta p_1)\right)}{2P}, \tag{39}$$

and

$$u_{11}(x,t) = a_0 + \frac{3}{2}\left(M + 2\coth(\zeta p_1)\right)^2 p_1^2 - \frac{\left(M+2\coth(\zeta p_1)\right)a_1p_1}{2P}, \tag{40}$$

In the same manner case III, results in the solution

$$u_{12}(x,t) = a_0 + \frac{3}{2}\left(M + \frac{\sinh(\zeta)+\sqrt{r^2-1}}{r+\cosh(\zeta)}\right)^2 - \frac{a_1\left(M+\frac{\sinh(\zeta)+\sqrt{r^2-1}}{r+\cosh(\zeta)}\right)}{2P}, \tag{41}$$

where $p_1 = 1$,

For case IV, the solution form is

$$u_{13}(x,t) = \frac{3}{2}(M + \coth(\zeta) + \operatorname{csch}(\zeta))^2 + \frac{a_1(M+\coth(\zeta)+\operatorname{csch}(\zeta))}{2P} + a_0, \tag{42}$$

with $p_1 = 1$.

3.2 Example 2. Burgers-Fisher equation

Consider Burgers-Fisher equation [24,25].

$$u_t = u_{xx} + uu_x + u(1-u). \tag{43}$$

Apply the transformation $u(x,t) = U(\zeta), \zeta = x - \lambda t$ to Eq. (43) Then it is reduced to the following ordinary differential equation:

$$-U + U^2 - \lambda U' + UU' - U'' = 0. \tag{44}$$

Balancing U'' with UU' yields m=1. Therefore, we are looking for the solution in the form

$$U = a_0 + b_0 + a_1\omega + b_1(1+\omega)^{-1}, \tag{45}$$

substituting Eqs. (45) and (4) in Eq. (44), we get a polynomial equation ω. Hence, equating the coefficient of ω^j (i.e., $j = 0,1,2,\ldots$) to zero and solving the obtained system of overdetermined algebraic equations using the symbolic manipulation package MATHEMATICA, results in :

$$M = 2P+1, k = P+1, a_1 = 0, P \neq 0, b_1 = \frac{a_0}{P}, b_1 \neq 0, \lambda = -a_0 + Pb_1 + 2. \tag{46}$$

For the first set, as in the previous example , we apply the compatibility condition, in using the solutions satisfying cases II & III & IV.

$$Pk = \frac{M^2 - p_1^2}{4}. \tag{47}$$

Therefore, substitute for P and k, from Eq. (46), into Eq. (47) and solve for p_1. It is found that

$$p_1 = 1 \quad \text{or} \quad p_1 = -1. \tag{48}$$

Therefore, the solution to the equation of the type (43), will be

$$u_1(x,t) = a_0\left(1 - \frac{1}{1+2\tanh(x-\lambda t)}\right), \tag{49}$$

and

$$u_2(x,t) = a_0\left(1 - \frac{1}{1+2\coth(x-\lambda t)}\right), \tag{50}$$

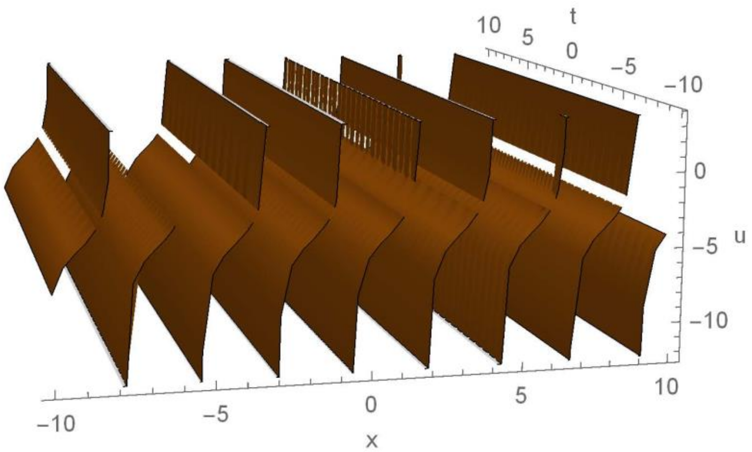

Figure 3. 3D and contour plots of the solution (51) with $a_0 = 0.03$ $P = 1$ and $r =$

In the same manner, case III results in the solution

$$u_3(x,t) = \frac{\left(-r - \cosh(\zeta) + \sinh(\zeta) + \sqrt{r^2-1}\right)a_0}{r + \cosh(\zeta) + \sinh(\zeta) + \sqrt{r^2-1}}, \tag{51}$$

with the condition that $p_1 = 1$.

For case IV, the solution form is

$$u_4(x,t) = \frac{(4P + \coth(\zeta) + \operatorname{csch}(\zeta) + 3)a_0}{4P + \coth(\zeta) + \operatorname{csch}(\zeta) + 1}, \tag{52}$$

with $p_1 = 1$,

4. Conclusion

In summary, an extended homogeneous balance method with computerized symbolic computation is developed to deal with nonlinear partial differential equations (PDEs). Traveling wave solutions were formally derived for Fisher's equation and Burgers-Fisher equation. This method can be also applied to other nonlinear evolution equations.

Conflict of interest

The authors declare no conflict of interest.

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