A Unique Common Coupled Fixed Point Theorem for Four Maps in Partial b-Metric-Like Spaces

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ABSTRACT: We prove the existence of a unique common coupled fixed point theorem for four mappings satisfying a general contractive condition on partial b-metric-like spaces. We also give an example to illustrate our main theorem. Our theorem generalizes and improves the theorem of [1].

Keywords: b-Metric-like space; Coupled fixed point; w-Compatibility maps.

1. Introduction and Preliminaries

The concept of b-metric space was introduced by Czerwik [2] as follows:

Definition 1.1 [2]: A b-metric on a non-empty set X is a function d : X × X → [0, ∞) such that for all x, y, z ∈ X and a constant k ≥ 1 the following three conditions hold true:

(i) d(x, y) = 0 if and only if x = y,
(ii) d(x, y) = d(y, x),
(iii) d(x, y) ≤ k[d(x, z) + d(z, y)] .

The triad (X, d, k) is called a b-metric space.

Alghamdi et al. [3] introduced the concept of b-metric-like spaces and proved some fixed point theorems for a single map.

Definition 1.2 [3]: A b-metric-like on a non-empty set X is a function d : X × X → [0, ∞) such that for all x, y, z ∈ X and a constant k ≥ 1 the following three conditions hold true:

(i) d(x, y) = 0 implies x = y,
(ii) d(x, y) = d(y, x),
(iii) d(x, y) ≤ k[d(x, z) + d(z, y)] .

The triad (X, d, k) is called a b-metric-like space.

Mathews [4] introduced the concept of a partial metric space as follows:

Definition 1.3 [4]: A mapping p : X × X → [0, ∞), where X is a non-empty set, is said to be a partial metric on X if for any x, y, z ∈ X the following are satisfied:

(i) x = y if and only if p(x, x) = p(x, y) = p(y, y),
(ii) p(x, y) ≤ p(x, y),
(iii) p(x, y) = p(y, x),
(iv) \( p(x, y) \leq p(x, z) + p(z, y) - p(z, z) \).

The pair \((X, p)\) is called a partial metric space.

Now we give the following definition by combining the Definitions 1.2 and 1.3.

**Definition 1.4:** A partial b-metric-like on a non-empty set \(X\) is a function \(p : X \times X \to [0, \infty)\) such that for all \(x, y, z \in X\) and a constant \(k \geq 1\) the following are satisfied:

\begin{enumerate}
    \item \( p(x, y) = 0 \) implies \( x = y \),
    \item \( p(x, x) \leq p(x, y) \), \( p(y, y) \leq p(x, y) \),
    \item \( p(x, y) = p(y, x) \),
    \item \( p(x, y) \leq k[p(x, z) + p(z, y) - p(z, z)] \).
\end{enumerate}

The triad \((X, p, k)\) is called a partial b-metric-like space.

**Definition 1.5:** Let \((X, p, k)\) be a partial b-metric-like space and let \(\{x_n\}\) be a sequence in \(X\) and \(x \in X\). The sequence \(\{x_n\}\) is said to be convergent to \(x\) if

\[ \lim_{n \to \infty} p(x_n, x) = p(x, x). \]

**Definition 1.6:** Let \((X, p, k)\) be a partial b-metric-like space.

(i) A sequence \(\{x_n\}\) in \((X, p, k)\) is said to be a Cauchy sequence if

\[ \lim_{n, m \to \infty} p(x_n, x_m) \]

exists and is finite.

(ii) A partial b-metric-like space \((X, p, k)\) is said to be complete if every Cauchy sequence \(\{x_n\}\) in \(X\) converges to a point \(x \in X\) so that

\[ \lim_{n, m \to \infty} p(x_n, x_m) = p(x, x) = \lim_{n \to \infty} p(x_n, x). \]

One can prove easily the following remark.

**Remark 1.7:** Let \((X, p, k)\) be a partial b-metric-like space and \(\{x_n\}\) be a sequence in \(X\) such that \(\lim_{n \to \infty} p(x_n, x) = 0\). Then

(i) \(x\) is unique,

(ii) \( \frac{1}{k} p(x, y) \leq \lim_{n \to \infty} p(x_n, y) \leq k p(x, y) \) for all \(y \in X\),

(iii) \( p(x_n, x_0) \leq k p(x_0, x_1) + k^2 p(x_1, x_2) + \cdots + k^{n-1} p(x_{n-2}, x_{n-1}) + k^n p(x_{n-1}, x_n) \) whenever \( \{x_n\} \in X \).

Let \((X, p, k)\) be a partial b-metric-like space and \(F, G : X \times X \) and \(f, g : X \to X\). For \(x, y, u, v \in X\), we denote

\[ M_{x, y}^{u, v} = \min \left\{ \frac{1}{2k} [p(fx, gu) + p(fy, gv)], \frac{1}{2k} [p(gu, F(x, y)) + p(gv, G(y, x))], \frac{1}{2k} [p(fx, G(u, v)) + p(fy, F(y, x))], \frac{1}{2k} [p(gu, F(x, y)) + p(gv, G(y, x))] \right\}. \]

and

\[ m_{x, y}^{u, v} = \max \left\{ \frac{1}{k} p(fx, gu), \frac{1}{k} p(fy, gv), \frac{1}{k} p(gu, F(x, y)), \frac{1}{k} p(gv, G(y, x)) \right\}. \]

Recently Bhaskar and Lakshmikantham \[5\] introduced the concept of coupled fixed point and discussed some problems of the uniqueness of a coupled fixed point and applied their results to the problems of the existence and uniqueness of a solution for the periodic boundary value problems. Later Lakshmikantham and Ciric \[6\] proved some coupled coincidence and coupled common fixed point results in partially ordered metric spaces.
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Definition 1.8 [6] An element \((x, y) \in \mathbb{X} \times \mathbb{X}\) is called
(i) a coupled coincident point of mappings \(F : \mathbb{X} \times \mathbb{X} \to \mathbb{X}\) and \(g : \mathbb{X} \to \mathbb{X}\) if \(gx = F(x, y)\) and \(gy = F(y, x)\).
(ii) a common coupled fixed point of mappings \(F : \mathbb{X} \times \mathbb{X} \to \mathbb{X}\) and \(g : \mathbb{X} \to \mathbb{X}\) if \(x = gx = F(x, y)\) and \(y = gy = F(y, x)\).

Definition 1.9 [7] The mappings \(F : \mathbb{X} \times \mathbb{X} \to \mathbb{X}\) and \(g : \mathbb{X} \to \mathbb{X}\) are called \(w\)-compatible if \(g(F(x, y)) = F(gx, gy)\) and \(g(F(y, x)) = F(gy, gx)\), whenever \(gx = F(x, y)\) and \(gy = F(y, x)\).

Recently, Abbas et al. [8] proved a common fixed point theorem for two maps of Jungck type satisfying generalized condition (B) in metric spaces (See Theorem 2.2, [8]). As a generalization of Theorem 2.2 of [8], Kaewcharoen et al. [1] obtained a common fixed point theorem for four maps satisfying a generalized condition in partial metric spaces.

In this paper, we obtain the existence of a unique common coupled fixed point theorem for four mappings satisfying a general contractive condition on partial b-metric-like spaces. We also give an example to illustrate our main theorem. Our theorem generalizes and improves the theorems of [1] and [8].

2. Main Result

Theorem 2.1: Let \((\mathbb{X}, p, k)\) be a complete partial b-metric-like space, \(F, G : \mathbb{X} \times \mathbb{X} \to \mathbb{X}\) and \(f, g : \mathbb{X} \to \mathbb{X}\) be mappings satisfying
\[
(2.1.1) \quad F(\mathbb{X} \times \mathbb{X}) \subseteq g(\mathbb{X}) \supseteq f(\mathbb{X}), \quad G(\mathbb{X} \times \mathbb{X}) \subseteq f(\mathbb{X}),
\]
\[
(2.1.2) \quad k p(F(x, y), G(u, v)) \leq \delta \frac{M_{x, y}}{m_{x, y}} + L
\]
for all \(x, y, u, v \in \mathbb{X}\), where \(\delta > 0\) and \(L \geq 0\), \(k < 1\), where \(L = \max \left\{ \frac{L}{1 - \delta} \right\} \).
\[
(2.1.3) \quad f(\mathbb{X}) \text{ or } g(\mathbb{X}) \text{ is closed},
\]
\[
(2.1.4) \quad \text{the pairs } (F, f), \text{ and } (G, g) \text{ are } w\text{-compatible}.
\]
Then \(F, G, f\) and \(g\) have a unique common coupled fixed point.

Proof. Let \((x_0, y_0) \in \mathbb{X} \times \mathbb{X}\). Since \(F(\mathbb{X} \times \mathbb{X}) \subseteq g(\mathbb{X})\), there exist \(x_1, y_1 \in \mathbb{X}\) such that \(gx_1 = F(x_0, y_0)\) and \(gy_1 = F(y_0, x_0)\). Since \(G(\mathbb{X} \times \mathbb{X}) \subseteq f(\mathbb{X})\), there exist \(x_2, y_2 \in \mathbb{X}\) such that \(fx_2 = G(x_1, y_1)\) and \(fy_2 = G(y_1, x_1)\). Continuing this process, we construct sequences \(\{x_n\}\) and \(\{y_n\}\) in \(\mathbb{X}\) such that
\[
gx_{2n+1} = F(x_{2n}, y_{2n}) = z_{2n},
gy_{2n+1} = F(y_{2n}, x_{2n}) = w_{2n},
fx_{2n+2} = G(x_{2n+1}, y_{2n+1}) = z_{2n+1},
fy_{2n+2} = F(y_{2n+1}, x_{2n+1}) = w_{2n+1}, \quad n = 0, 1, 2, 3, \ldots
\]
Now consider
\[
p(z_{2n}, z_{2n+1}) \leq k p(F(x_{2n}, y_{2n}), G(x_{2n+1}, y_{2n+1}))
\]
\[
\leq \delta M_{x_{2n}, y_{2n}}^{z_{2n}, z_{2n+1}} + L m_{x_{2n}, y_{2n}}^{z_{2n}, z_{2n+1}}
\]
where
\[
M_{x_{2n}, y_{2n}}^{z_{2n}, z_{2n+1}} = \max \left\{ \begin{array}{l}
p(z_{2n-1}, z_{2n}), \quad p(w_{2n-1}, w_{2n}), \quad p(z_{2n-1}, z_{2n}), \quad p(w_{2n-1}, w_{2n}), \quad p(z_{2n-1}, z_{2n}), \\
\frac{1}{2k}[p(z_{2n-1}, z_{2n}) + p(z_{2n}, z_{2n})], \quad \frac{1}{2k}[p(w_{2n-1}, w_{2n}) + p(w_{2n}, w_{2n})]
\end{array} \right\}
\]
\[
\leq \max \left\{ \begin{array}{l}
p(z_{2n-1}, z_{2n}), \quad p(w_{2n-1}, w_{2n}), \quad p(z_{2n-1}, z_{2n}), \\
p(z_{2n}, z_{2n}), \quad p(w_{2n}, w_{2n})
\end{array} \right\}
\]
from \(k \geq 1\) and from \((p_4)\).
\[ m_{z_{m}, z_{m}} = \min \{ \frac{1}{k} \left( \begin{array}{c}
p(z_{2n}, z_{2n}), p(w_{2n}, w_{2n}) \\
p(z_{2n-1}, z_{2n-1}), p(w_{2n-1}, w_{2n-1}) \\
n - n - \end{array} \right) \} \]

\[ \leq \min \{ \sum_{i=1}^{m} \left( \begin{array}{c}
p(z_{2n}, z_{2n}), p(w_{2n}, w_{2n}) p(z_{2n}, z_{2n}) \\
p(z_{2n-1}, z_{2n-1}), p(w_{2n-1}, w_{2n-1}) + p(z_{2n-1}, z_{2n-1}) \\
n - n - \end{array} \right) \} \]

\[ = \min \{ p(z_{2n}, z_{2n}), p(w_{2n}, w_{2n}) \} \text{ from (p3)} \]
\[ \leq \max \{ p(z_{2n}, z_{2n}), p(w_{2n}, w_{2n}) \} \]
\[ \leq \max \{ p(z_{2n}, z_{2n}), p(w_{2n}, w_{2n}) \} \text{ from (p3).} \]

Thus
\[ p(z_{2n}, z_{2n}) \leq \delta \max \left\{ p(z_{2n}, z_{2n}), p(w_{2n}, w_{2n}) + L \max \{ p(z_{2n}, z_{2n}), p(w_{2n}, w_{2n}) \} \right\} \]

Similarly
\[ p(w_{2n}, w_{2n}) \leq \delta \max \left\{ p(z_{2n}, z_{2n}), p(w_{2n}, w_{2n}) + L \max \{ p(z_{2n}, z_{2n}), p(w_{2n}, w_{2n}) \} \right\} \]

Thus
\[ \max \left\{ p(z_{2n}, z_{2n}), p(w_{2n}, w_{2n}) \right\} \leq \delta \max \left\{ p(z_{2n}, z_{2n}), p(w_{2n}, w_{2n}) + L \max \{ p(z_{2n}, z_{2n}), p(w_{2n}, w_{2n}) \} \right\} \]

If
\[ \max \left\{ p(z_{2n}, z_{2n}), p(w_{2n}, w_{2n}) \right\} \leq \max \left\{ p(z_{2n}, z_{2n}), p(w_{2n}, w_{2n}) \right\}, \]

then from (2)
\[ \max \{ p(z_{2n}, z_{2n}), p(w_{2n}, w_{2n}) \} \leq \frac{L}{1 - \delta} \max \{ p(z_{2n}, z_{2n}), p(w_{2n}, w_{2n}) \}. \]

If
\[ \max \left\{ p(z_{2n}, z_{2n}), p(w_{2n}, w_{2n}) \right\} \leq \max \left\{ p(z_{2n}, z_{2n}), p(w_{2n}, w_{2n}) \right\}, \]

then from (2)
\[ \max \{ p(z_{2n}, z_{2n}), p(w_{2n}, w_{2n}) \} \leq (\delta + L) \max \{ p(z_{2n}, z_{2n}), p(w_{2n}, w_{2n}) \}. \]

Hence
\[ \max \{ p(z_{2n}, z_{2n}), p(w_{2n}, w_{2n}) \} \leq l \max \{ p(z_{2n}, z_{2n}), p(w_{2n}, w_{2n}) \} \]

where
\[ l = \max \left\{ \frac{L}{1 - \delta} + L \right\} < 1. \]

Similarly we can show that
\[ \max \{ p(z_{2n-1}, z_{2n-1}), p(w_{2n-1}, w_{2n-1}) \} \leq l \max \{ p(z_{2n-1}, z_{2n-1}), p(w_{2n-1}, w_{2n-1}) \}. \]

Hence
\[ \max \{ p(z_{n}, z_{n}), p(w_{n}, w_{n}) \} \leq l \max \{ p(z_{n-1}, z_{n-1}), p(w_{n-1}, w_{n-1}) \}, \quad n = 1, 2, 3, \ldots \]

Thus
\[ \max \{ p(z_{n}, z_{n}), p(w_{n}, w_{n}) \} \leq l^h \max \{ p(z_{n}, z_{n}), p(w_{n}, w_{n}) \}. \]

From (3), it follows that
\[ \lim_{n \to \infty} p(z_{0}, z_{n}) = 0 = \lim_{n \to \infty} p(w_{0}, w_{n}). \]

For \( m > n \), consider
\[ \max \{ p(z_{m}, z_{m}), p(w_{m}, w_{m}) \} \]

\[ \leq \max \left\{ \begin{array}{c}k p(z_{m}, z_{m}) + k^2 \max \{ p(z_{m}, z_{m}), p(w_{m}, w_{m}) \} \\
+ \ldots + k^{m-1} \max \{ p(z_{m}, z_{m}), p(w_{m}, w_{m}) \} \\
k^{m-1} \max \{ p(z_{m}, z_{m}), p(w_{m}, w_{m}) \} \end{array} \right\} \]

\[ \leq k \max \{ p(z_{m}, z_{m}), p(w_{m}, w_{m}) \} + k^2 \max \{ p(z_{m}, z_{m}), p(w_{m}, w_{m}) \} \]

\[ + \ldots + k^{m-1} \max \{ p(z_{m}, z_{m}), p(w_{m}, w_{m}) \} + k^{m-1} \max \{ p(z_{m}, z_{m}), p(w_{m}, w_{m}) \} \]
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\[
\leq (kt^n + k^2 t^{n+1} + \ldots + k^{m-n-1} t^{m-2} + k^{m-n-1} t^{m-1}) \max \left\{ \frac{p(z_n, z_i)}{p(w_n, w_i)} \right\} \leq k^n \left( 1 + k t^1 + k^2 t^2 + \ldots + k^{m-n-2} t^{m-n-2} + k^{m-n-2} t^{m-n-1} \right) \max \left\{ \frac{p(z_n, z_i)}{p(w_n, w_i)} \right\} \\
v_k \left( 1 + k t^1 + k^2 t^2 + \ldots + k^{m-n-2} t^{m-n-2} + k^{m-n-2} t^{m-n-1} \right) \max \left\{ \frac{p(z_n, z_i)}{p(w_n, w_i)} \right\} \leq \frac{k t^1}{1 - k t} \max \left\{ \frac{p(z_n, z_i)}{p(w_n, w_i)} \right\}, \text{ since } k t < 1.
\]

Hence

\[
\lim_{n \to \infty} p(z_n, z_m) = 0 = \lim_{n \to \infty} p(w_n, w_m).
\]

Thus \( \{z_n\} \) and \( \{w_n\} \) are Cauchy in \((X, p, k)\).

Since \( X \) is complete, the sequences \( \{z_n\} \) and \( \{w_n\} \) converge to some \( \alpha \) and \( \beta \) in \( X \) respectively such that

\[
\lim_{n \to \infty} p(z_n, z) = p(\alpha, \alpha) = 0 = \lim_{n \to \infty} p(w_n, w) = p(\beta, \beta).
\]

Now from (5), we have

\[
 p(\alpha, \alpha) = 0 = p(\beta, \beta).
\]

Suppose \( f(X) \) is closed. Since \( f(x_{2n+2}) = z_{2n+1} \to \alpha \) and \( f(y_{2n+2}) = w_{2n+1} \to \beta \), it follows that \( \alpha = fu \) and \( \beta = fv \) for some \( u, v \in X \).

Consider

\[
0, 0, p(\alpha, \beta) \leq \delta \max \{p(\alpha, F(u, v)), p(\beta, F(v, u))\}, \quad \text{from (4) and Remark 1.7 (ii)}
\]

Thus

\[
p(\alpha, F(u, v)) \leq \delta \max \{p(\alpha, F(u, v)), p(\beta, F(v, u))\}.
\]

Similarly we can show that

\[
p(\beta, F(v, u)) \leq \delta \max \{p(\alpha, F(u, v)), p(\beta, F(v, u))\}.
\]

Hence

\[
\max \{p(\alpha, F(u, v)), p(\beta, F(v, u))\} \leq \delta \max \{p(\alpha, F(u, v)), p(\beta, F(v, u))\},
\]

which in turn yields that \( \alpha = F(u, v) \) and \( \beta = F(v, u) \).

Thus \( fu = \alpha = F(u, v) \) and \( fv = \beta = F(v, u) \).

Since the pair \((F, f)\) is \(w\)-compatible, we have

\[
fu = F(\alpha, \beta) \quad \text{and} \quad fv = F(\beta, \alpha).
\]

Since \( \alpha = F(u, v) \in F(X \times X) \subseteq g(X) \), there exists \( r \in X \) such that \( \alpha = gr \).

Since \( \beta = F(v, u) \in F(X \times X) \subseteq g(X) \), there exists \( t \in X \) such that \( \beta = gt \).
Now \( p(\alpha, G(r, t)) \leq s p(F(u, v), G(r, t)) \leq \delta \) \( M_{r,t}^{u,v} + L \ m_{r,t}^{u,v} \)

\[
M_{r,t}^{u,v} = \max \left\{ \frac{1}{2k} [p(fu, gr) + p(gr, F(u, v))], \frac{1}{2k} [p(fv, G(t, r)) + p(gt, F(v, u))] \right\}
\]

\[
m_{r,t}^{u,v} = 0.
\]

Thus \( p(\alpha, G(r, t)) \leq \delta \max \{ p(\alpha, G(r, t)), p(\beta, G(t, r)) \} \).

Similarly we can show that \( p(\beta, G(t, r)) \leq \delta \max \{ p(\alpha, G(r, t)), p(\beta, G(t, r)) \} \).

Hence \( \max \{ p(\alpha, G(r, t)), p(\beta, G(t, r)) \} \leq \delta \max \{ p(\alpha, G(r, t)), p(\beta, G(t, r)) \} \) which in turn yields that \( gr = \alpha = G(r, t) \) and \( gt = \beta = G(t, r) \).

Since the pair \((G, g)\) is w-compatible, we have \( ga = G(\alpha, \beta) \) and \( gb = G(\beta, \alpha) \).

Now consider

\[
p(f_\alpha, \alpha) \leq k p(F(\alpha, \beta), G(r, t)) \leq \delta \) \( M_{r,t}^{\alpha,\beta} + L \ m_{r,t}^{\alpha,\beta} \)
\]

\[
M_{r,t}^{\alpha,\beta} = \max \left\{ \frac{1}{2k} [p(f_\alpha, gr) + p(gr, F(\alpha, \beta))], \frac{1}{2k} [p(tf_\beta, G(t, r)) + p(gt, F(\beta, \alpha))] \right\}
\]

\[
m_{r,t}^{\alpha,\beta} = 0.
\]

Thus \( p(f, \alpha) \leq \delta \max \{ p(f, \alpha), p(f, \beta) \} \).

Similarly we can show that \( p(f, \beta) \leq \delta \max \{ p(f, \alpha), p(f, \beta) \} \).

Hence \( \max \{ p(f, \alpha), p(f, \beta) \} \leq \delta \max \{ p(f, \alpha), p(f, \beta) \} \) which in turn yields that \( fa = \alpha = f \) and \( fb = \beta \).

Similarly we can show that \( ga = \alpha \) and \( gb = \beta \).

Thus
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F (α, β) = fα = α = gα = G(α, β) and
F (β, α) = fβ = β = gβ = G(β, α).
Hence (α, β) is a common coupled fixed point of F, G, f and g. Uniqueness of this common coupled fixed point follows easily from (2.1.2).

Now, we give an example to illustrate our main Theorem 2.1.

Example 2.2 Let X = [0, 1] and p(x, y) = max{x^2, y^2}. Then (X, p, k) is a complete partial b-metric-like space with k = 2. Define F, G : X × X → X and f, g : X → X as F(x, y) = 0, G(x, y) = 4x^4, fx = 2x^2 and gx = x. Then

\[ k \frac{p(F(x, y), G(u, v))}{p(gu, G(u, v))} = \frac{1}{8} \leq \frac{1}{8} + \frac{1}{4} M_{x,y} + 0 M_{x,y}. \]

Here \( \delta = \frac{1}{8} \), k = 2, \( \frac{l}{4} = \frac{1}{4} < 1 \). Clearly (2.1.1), (2.1.3) and (2.1.4) are satisfied and (0, 0) is the unique common coupled fixed point of F, G, f and g.

Theorem 2.1 is a generalization and improvement of the following:

Theorem 2.3 (Theorem 2.1, [1]): Let (X, p) be a complete partial metric space. Suppose that f, g, F, G : X → X satisfying the following conditions

(2.3.1) \( f(X) \subseteq g(X) \) and \( F(X) \subseteq G(X) \),
(2.3.2) there exist \( \delta > 0 \) and \( L \geq 0 \) with \( \delta + 2L < 1 \) such that
\[ p(Fx, fy) \leq \delta M(x, y) + L \min\{p(gx, Fx), p(Gy, fy), p(gx, fy), Gy, Fx\} \]
for all \( x, y \in X \), where
\[ M(x, y) = \max\{p(gx, Gy), p(gx, Fx), p(Gy, fy), \frac{1}{2} [p(gx, fy) + p(Gy, Fx)]\}, \]
(2.3.3) \( f(X) \) or \( g(X) \) is closed and
(2.3.4) the pairs \( (f, G) \) and \( (g, F) \) are w-compatible.

Then f, g, F and G have a unique common fixed point in X.

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