

An Enhanced Smith Predictor Based Control System Using Feedback-feedforward Structure for Time-delay Processes

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Received 17 March 2017; Accepted 1 October 2017

Abstract: This paper presents enhanced Smith predictor based control systems (SPBCSs) for processes with a time-varying or fixed time delay. This work focuses on improving the arrangement and asynchrony of SPBCS components rather than the design of the predictor and the feedback controller, which have been well discussed in the literature. The proposed control system advances SPBCS through implementation of two design recommendations: (i) replacing the classical feedback controller by a feedback-feedforward control system, and (ii) synchronizing the reference signal and the predicted output. As a result, common shortcomings of SPBCSs or control systems based on Pade approximation, *i.e.* the trade-off between performance and steady-state error, and instability associated with non-minimum-phase systems do not exist in the proposed SPBCS. The superior performance of the proposed control system is demonstrated with two examples: temperature control of an infrared dryer (a system with fixed time-delay) and air-fuel ratio of a lean-burn spark-ignition engine (a system with time-varying delay and lag). The proposed control system is shown to clearly outperform the conventional SPBCS and Internal Model Control (IMC) PID based on Pade approximation for aforementioned examples and performs satisfactorily in the presence of noises, actuator saturations, and severe model inaccuracies.

Keywords: Smith predictor; Time delay system; Dryer; Engine.

نظام سميث المحسن للتنبؤ القائم على نظام المراقبة باستخدام جهاز التغذية الراجعة -التغذية المرتدة لعمليات تأخير الوقت

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المخلص: تعرض هذه الورقة البحثية أنظمة سميث المحسنة للتنبؤ القائم على نظام المراقبة (SPBCSs) للعمليات التي بها تأخير زمني متغير أو ثابت. ويركز هذا العمل على تحسين الترتيب وعدم التزامن لمكونات SPBCSs بدلا من تصميم نظام التنبؤ ومراقبة التغذية الراجعة التي تم مناقشتها بشكل جيد في الأدبيات. ويقدم نظام المراقبة المقترح نظام سبب من خلال تنفيذ توصيتين للتصميم هما: (1) الاستعاضة عن نظام مراقبة التغذية الراجعة الكلاسيكية بنظام مراقبة التغذية المسبقة و (2) مزامنة الإشارة المرجعية والنتائج المتوقعة. ونتيجة لذلك، فإن أوجه القصور المشتركة في نظام (SPBCSs) أو أنظمة المراقبة القائمة على التقديرات التقريبية لنظام بايد. أي المقايضة بين الأداء وخطأ الحالة الثابتة، وعدم الاستقرار المقترن بالنظم غير الدنيا، لا توجد في نظام (SPBCSs) المقترح. ويتجلى الأداء المتفوق لنظام المراقبة المقترح في مثالين هما: مراقبة درجة الحرارة في مجفف الأشعة تحت الحمراء (نظام تأخير زمني ثابت) ونسبة وقود الهواء الناتج عن اشتعال شرارة الاحتراق الضعيفة للمحرك (نظام ذو متغيرات زمنية في التأخير والتباطؤ). وقد تبين أن نظام التحكم المقترح يتفوق بشكل واضح على نظام (SPBCSs) التقليدي و نظام التحكم الداخلي (IMC-PID) القائم على التقديرات التقريبية للأمثلة المذكورة آنفا، كما أنه يقوم بإدائه بشكل مرض في وجود الضوضاء والتشبع في المحركات والقصور الحاد في دقة النموذج.

الكلمات المفتاحية: تنبؤ سميث، نظام تأخير الوقت، المجفف، المحرك

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1. Introduction

A wide variety of dynamic systems witness dead-time or delay. So called time-delay systems (Zhong 2006; Wu *et al.* 2010) include chemical processes, engines, manufacturing, micro-aerial vehicles and telecommunication systems, etc. (Liu 2014; Sanz *et al.* 2017). Time-delay is generally regarded as a main source of instability and poor performance in control systems (Li and deSouza 1997). In order to effectively address this control issue, the difference between two categories of time-delay systems needs to be recognised. In the first category, states of a system influence their time derivative with a delay. This type of delay is mainly network-induced, *i.e.* it is resulted by the time needed to exchange data between devices in electrical/communication networks. The control of this type of time-delay is addressed using Lyapunov-Krasovskii and Razumikhin theorems (Wu *et al.* 2010) and is beyond the scope of process control and this paper. In the second category, the control inputs influence the time derivative of the system states with a delay, *i.e.* a noticeable time is needed for the actuator(s) to affect the states and the output (Zhong 2006). This type of time-delay is a prevalent concern in process control (Bequette 2008) and is studied in this paper.

Two main approaches have been employed to control the second category of time-delay systems: Pade approximation and predictive methods. In the first approach, the term presenting the delay is approximated using Pade technique (Bequette 2008). Then, the resultant model is used in control system design. Pade approximation results in model inaccuracy and transforms a stable minimum-phase system to a non-minimum-phase one with a higher order (Ebrahimi *et al.* 2012). On the other hand, a predictive method, such as a Smith predictor (Smith 1957), does not add the aforementioned restrictions; hence, it is expected to present higher performance. Smith predictors were originally used for stable SISO systems, but further research extended their application to unstable systems (Kwak *et al.* 1999, Mataušek and Ribić 2012) and systems with long delays (Astrom *et al.* 1994). As a result, Smith predictors were suggested to be a part of control solution for any stable or unstable time-delay system (DePoar 1985; Zhong 2006).

Despite all the advantages and advances of SPBCSs, there are reports showing the superiority of feedback control systems based on Pade approximation over SPBCSs. For example, in the control of linear systems with identifiable time-varying delays (Ebrahimi *et al.* 2014a; Ebrahimi *et al.* 2014b; Mohammadzaheri *et al.* 2015a and 2015b). This research investigates and identifies two reasons behind relatively poor performance of SPBCSs and proposes two design recommendations to resolve the identified issues. Unlike most of the literature on SPBCSs, the focus of this research is on the structure of SPBCSs rather than the design of the predictor or the feedback controller. The proposed control system, an enhanced SPBCS, is assessed on systems with fixed or time-varying delays. A thorough assessment shows the superior performance of the proposed control system.

2. Shortcomings of SPBCS

Figure 1 shows a typical Smith-predictor-based control system (SPBCS), where $G(z)$ and $\hat{G}(z)$ are the plant original and approximated transfer functions without the delay, respectively, r is the delay, and $C(z)$ is a linear classical controller designed for $G(z)$. Regardless of the methods used to design the predictor and the classical controller, this structure has two main shortcomings causing a relatively low performance:

- (i) Design of a feedback classical controller (a vital part of SPBCS) is always a trade-off between performance and steady-state error. Moreover, windup phenomenon (the influence of actuator's saturation on integrals (Aryan *et al.* 2010)) needs to be compensated. These limits influence the performance of the SPBCS; where other methods, *e.g.* the ones based on Pade approximation, may use non-classical controllers (*e.g.* sliding mode (Ebrahimi *et al.* 2014b)) with no trade-off or windup issues.
- (ii) Figure 2, a re-arrangement of Fig. 1, demonstrates that the reference $y_d(k)$ is actually compared with the estimated output at a different time, $\hat{y}(k+r)$, *i.e.* there is an asynchrony in conventional SPBCS.

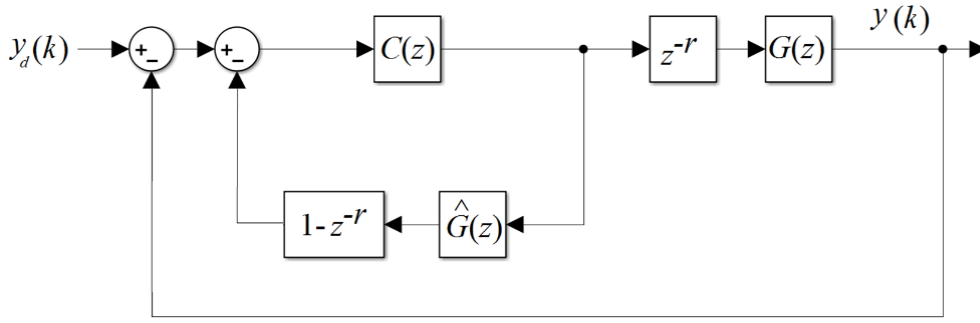


Figure 1. A conventional Smith-predictor-based control system.

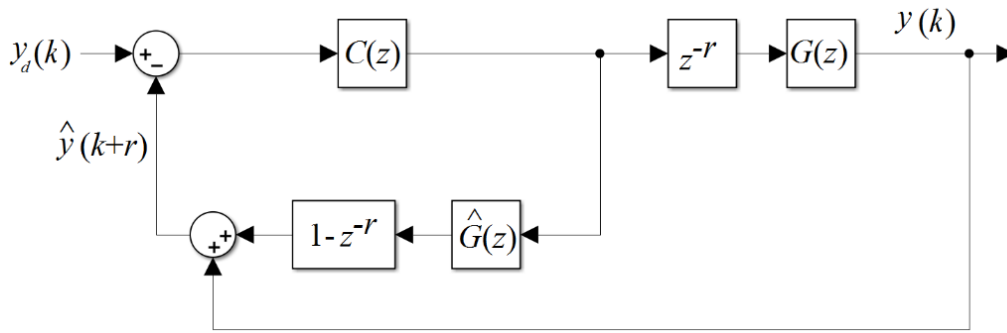


Figure 2. Another representation of the common Smith-predictor-based control system shown in Fig. 1, assuming the discrepancy of $G(z)$ and $\hat{G}(z)$ is negligible.

Both aforementioned shortcomings are resolved in the proposed structure of SPBCS.

3. Methodology

The following design elements are recommended to address the shortcomings listed in the previous section.

Recommendation 1: A feedback-feedforward control law replaces the feedback controller of $C(z)$ in Fig. 2 (which is designed for the plant without delay or $G(z)$). The recommended control system is proposed in the following Corollary. This removes trade-off and windup issues.

Corollary: A process modelled by (1) or its discrete equivalent, is asymptotically stabilized by a control law of (2) if condition (3) is satisfied:

$$G(s) = \frac{b}{Ts+1}, \quad (1)$$

$$u(t) = Ke(t) + u^*(t), \quad (2)$$

$$\frac{bK+1}{T} > 0, \quad (3)$$

where $e(t) = y_d(t) - y(t)$. Subscript d refers to desired status ($y = y_d, \dot{y} = \dot{y}_d$) and $u^*(t)$ is derived from (4):

$$T\dot{y}_d(t) + y_d(t) = bu^*(t). \quad (4)$$

Proof is in the Appendix.

Accordingly, if, in (4), $b > 0$ and $T > 0$, with any positive K , stability is maintained, and steady-state error converges to zero. As a result, there is no trade-off between stability and steady-state error. Besides, there is no integral in (2); hence, windup does not exist. The sole source of performance restriction is the incapability to achieve high values of K due to actuator limits. However, the feedforward controller, which generates $u^*(t)$, is model-based. Hence,

inaccuracy in model identification may lead to loss of performance as a disadvantage of the employed feedback-feedforward structure; this issue has been investigated for the proposed design in example 2. It should be noted that the control systems with feedforward-feedback structure have been investigated and used for different purposes (De Luca 2000; Bresch-Pietri and Krstic 2009; Mohammadzaheri *et al.* 2009). In this paper, this structure is adopted in SPBCSs.

Recommendation 2: In tracking problems, *i.e.* where the reference, y_d , is subject to change, the value of reference signal at r instants ahead of current time, $y_d(k+r)$, is used both to generate feedforward control commands and to be compared with the output of the Smith predictor, as shown in Fig. 3. The value of reference signal is normally available upfront in process plants (Bresch-Pietri and Krstic 2009). This recommendation can be disregarded for regulation problems, where the reference is constant.

3.1 Example1: An Infrared Dryer, A System with Fixed Delay

Figure 4 shows an infrared dryer with a pair of lamps/thermocouples. Such dryers can be used to dry food and other bio products (Reed *et al.* 2000; Chuanzhu *et al.* 2005; Sharma *et al.* 2005; Roknul *et al.* 2014) or in a variety of other engineering applications (Mirsepahi *et al.* 2017). Using the experimental data and the least square method, the system transfer function has been approximated by (Mohammadzaheri *et al.* 2015a):

$$H_{dryer}(s) = \frac{52.7108}{1+152.812s} e^{-1.3s} \quad (5)$$

$$G_{c-imc-dryer}(s) = \frac{2.9114}{\lambda+0.65} + \left(\frac{0.0190}{\lambda+0.65} \right) \frac{1}{s} + \left(\frac{1.8844}{\lambda+0.65} \right) \quad (6)$$

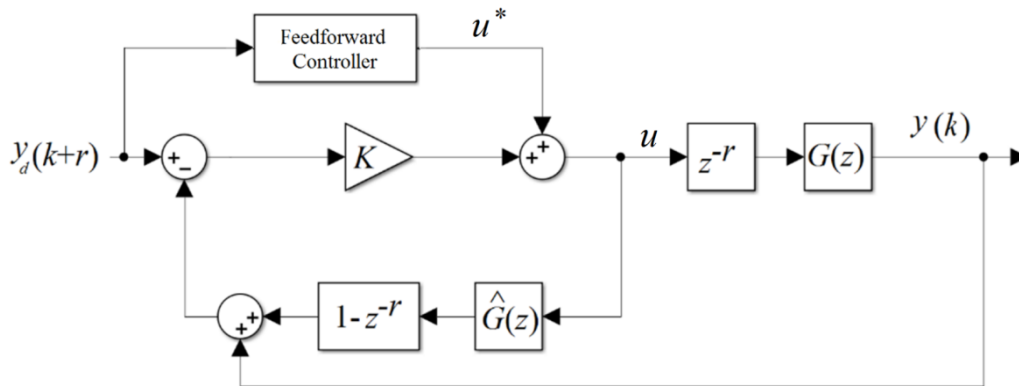


Figure 3. The proposed smith-predictor-based control system.

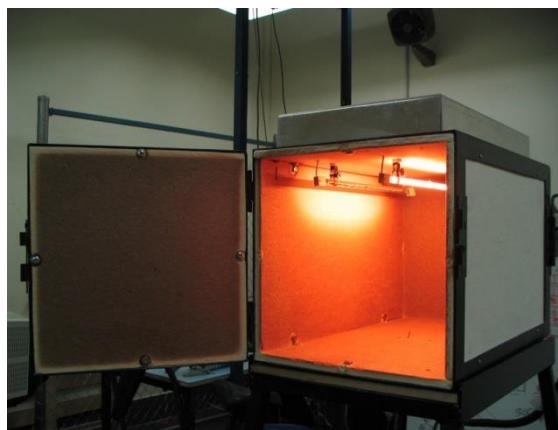


Figure 4. The infrared dryer.

The input is the control command (in the range of 0 and 10V) which excites the lamp power amplifier, and the output is the measured temperature in °C. Equation (5) evidently meets the condition of the proposed corollary, *i.e.* (3), with any $K > 0$.

Three control systems were examined on the dryer model:

- (i) A feedback PID controller, (6), designed through Pade approximation and internal model control (IMC), with the method detailed in (Bequette 2008):
- (ii) A conventional SPBCS (Fig. 1) with an IMC PI controller of (7) (a continuous equivalent of $C(z)$ in Fig. 1):

$$C_{dryer}(s) = \frac{2.8991}{\lambda} + \frac{0.0190}{\lambda} \frac{1}{s}, \quad (6)$$

where $\lambda=1$ was chosen empirically for both (6) and (7).

- (iii) The proposed enhanced SPBCS (Fig. 3) with feedforward component of $u^*=0.0190 y_d$, derived for the time-delay process presented by (5) at the steady state desired

situation of $\dot{y}_d = 0$. With such a feedforward component, the condition presented by (3) is satisfied for any positive feedback gain of K . A feedback gain of $K = 1$ was chosen empirically to maintain reasonable trade-off between fast response and sensitivity to noise.

The mean of absolute controller error (MACE) was used as the control performance criterion:

$$MACE = \frac{\int_{t_i}^{t_f} |y_d(i) - y(i)| dt}{t_f - t_i} \quad (7)$$

where t_f and t_i are final and start times of operation. Figure 5 and Table 1 compare the performance of three controllers with and without a random noise in the range of $[-3 +3]^\circ\text{C}$ and with the fixed sampling frequency of 100 Hz. Feedback PID IMC fails to control the system in presence of noise. The proposed SPBCS outperforms both conventional SPBCS and PID IMC feedback with 9% to 22% less MACE.

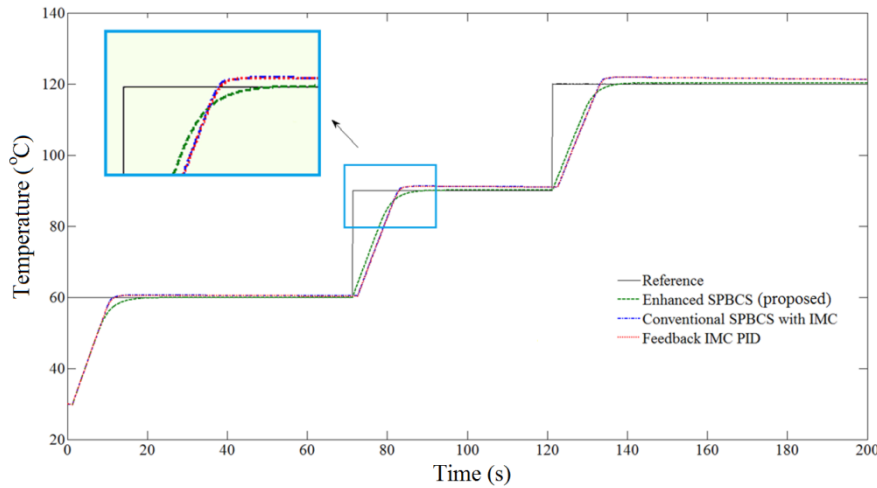


Figure 5. Performance of three control systems on an infrared dryer model without noise, where conventional IMC SPBCS and feedback IMC PID almost match.

Table 1. Mean of absolute control error for different controllers.

	MACE [°C]	
	Noise-less	With Noise
IMC PID	2.0068	Failure
Conventional IMC SPBCS	1.9911	2.2875
Enhanced SPBCS	1.5686	2.0898

3.2 Example 2 – Air to Fuel Ratio of a Lean-Burn Engine, a System with Time-Varying Delay and Lag

The second example is control of normalized air-fuel ratio (AFR) of a lean-burn spark-ignition (SI) engine. The importance of this control problem is on rise due to increasing concerns and new **regulations** about emissions (Pace and Zhu 2014; Lin *et al.* 2017) and fuel consumption (Ghods *et al.* 2017). Normalized AFR is defined as following (Zope *et al.* 2011).

$$AFR = \frac{1}{AFR_s} \frac{\dot{m}_a}{\dot{m}_f}, \quad (8)$$

where \dot{m}_a and \dot{m}_f are air and fuel mass flow rates, respectively, and AFR_s is the stoichiometric air-fuel ratio which is commonly 14.7 for gasoline engines (Ebrahimi *et al.* 2012). Input AFR to the engine can be calculated using measured fuel and air mass flow rates. The output AFR is measured by the universal exhaust gas oxygen (UEGO) sensor (Fig. 6). The input and output AFR relationship is approximated by a first-order transfer function (Zope *et al.* 2011).

$$H_{fuel\ path}(s) = \frac{1}{Ts + 1} e^{-\theta s}. \quad (9)$$

where lag (T) and time delay (θ) can be estimated as following:

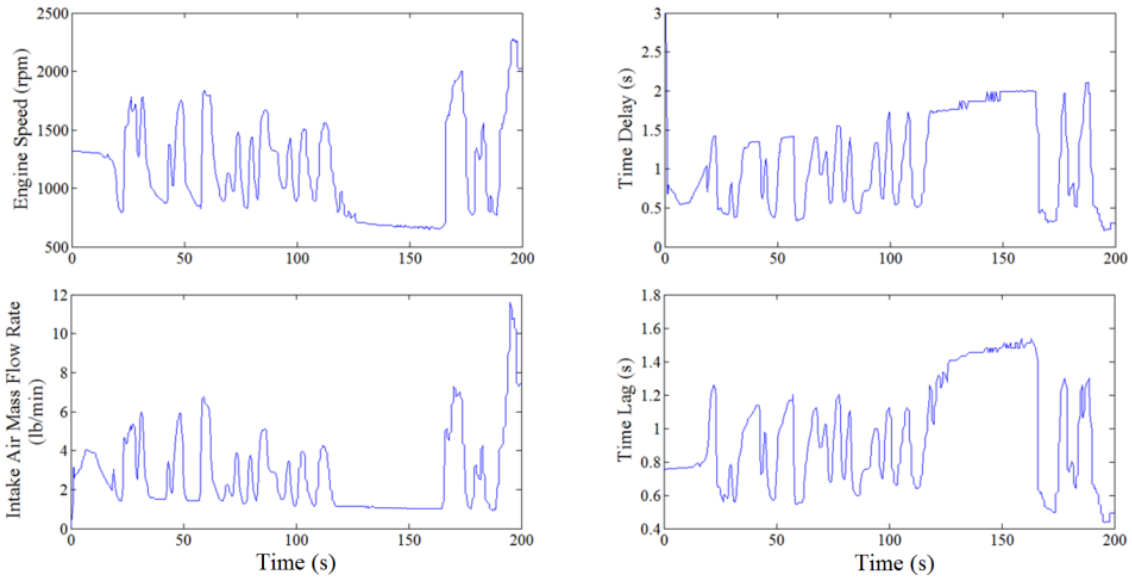


Figure 7. Engine speed and intake air flow mass rate according to a typical Federal Test Procedure (FTP) and time delay (θ) and lag time (T) calculated based on (11).

$$\begin{cases} T = \frac{\alpha}{N}, \\ \theta = \frac{120}{N} + \frac{\beta}{\dot{m}_a}, \end{cases} \quad (10)$$

where N is engine rotational speed, and α and β are coefficients which are identified empirically with the values of $\alpha = 997.4$ and $\beta = 0.0305 \text{ lb}$ for the investigated engine (Ebrahimi *et al.* 2012).

Figure 7 shows experimental data of engine speed and air mass flow as well as the time delay and lag time collected in a typical Federal Test Procedure (FTP) for an F-150 Ford truck with a V8 4.6L lean-burn SI engine (Ebrahimi *et al.* 2012).

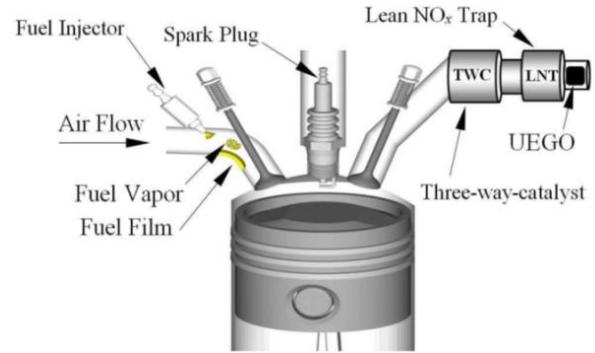


Figure 6. A schematic of air-fuel path in a lean-burn SI engine (Ebrahimi *et al.* 2014).

For the aforementioned engine and operating conditions, depicted in Fig. 7, a feedback IMC PID was designed for (10). Furthermore, an IMC PI controller was designed for (10) excluding the exponential term to be used in the conventional Smith arrangement depicted in Fig. 1. For these controllers, the values of λ were empirically found as 1.2 and 0.1, respectively. Equation (10) evidently meets the condition of the proposed corollary, *i.e.* (3), with any $K > 0$. Therefore, the enhanced SPBCS (depicted in Fig. 3) is applicable on this system. The feedforward control command is $u^*=y_d$, based on (4 and 10). A feedback gain of $K=1$ was also empirically chosen. A random noise in the range of $\pm 0.5\%$ of AFR_s (Fiengo *et al.* 2005), *i.e.* ± 0.0735 , was considered to assess the noise rejection of different control systems. A fixed sampling frequency of 12.5 Hz, which is achievable by

commercially available actuators (Woodward 2014) was also chosen. Figure 8 and Table 2 compare the performance of three investigated control systems with the control input saturation limits of 0 and 6.

In this example, the coefficients of α and β in (11) may be inaccurate due to imperfection of their identification process. In order to evaluate the effect of the inaccuracies, 20%, 30% and 50% of identification error were considered. Then, the control systems were designed based on inaccurate lag (T) and delay (θ) obtained from the erroneous α and β (Fig. 9). As demonstrated in Fig. 10 and Table 3, the proposed control system exhibits satisfactory performance with inaccurate α and β , where two other control systems witness either a significant drop in performance or total failure in most cases.

Table 2. Mean of absolute control error for different controllers on a lean-burn engine.

	MACE	
	Noise-less	With Noise
Feedback IMC PID	0.138	0.295
Conventional IMC SPBCS	0.122	0.286
Enhanced SPBCS	0.072	0.082

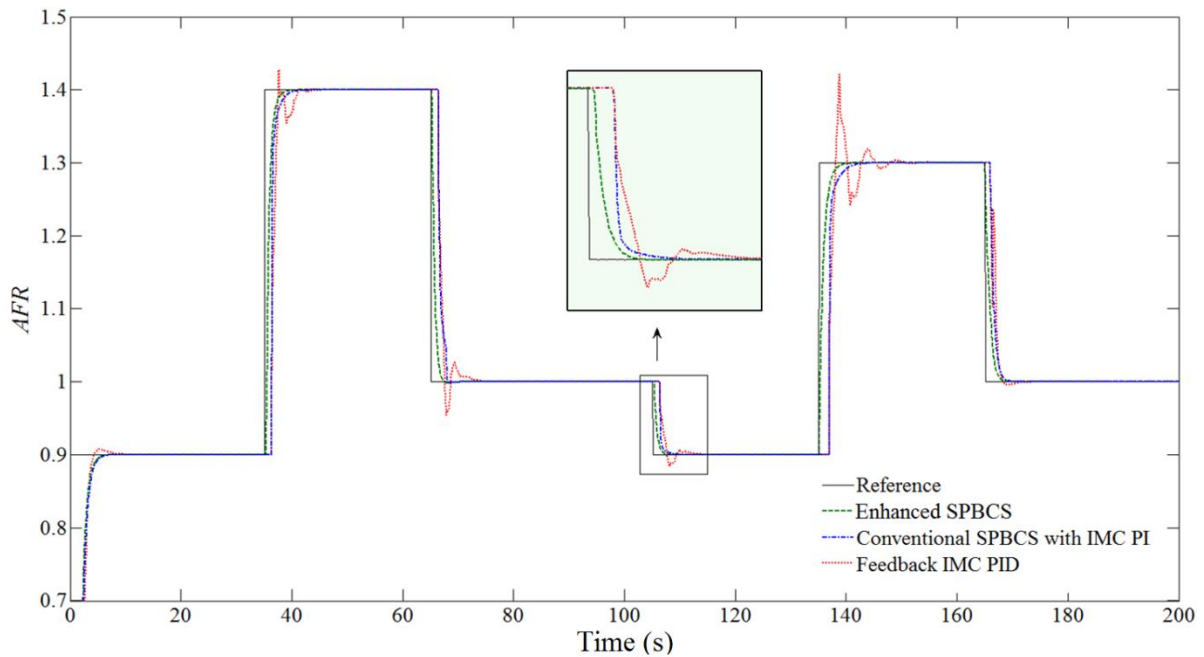


Figure 8. Performance of three control systems on an engine model without noise.

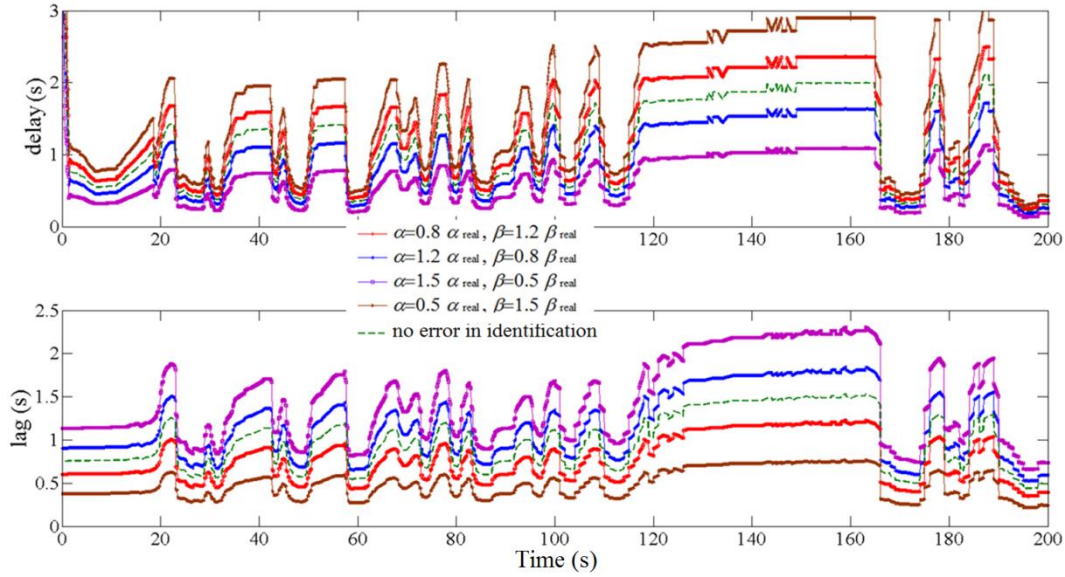


Figure 9. Delay and lag variations in the case of error in identification of α and β .

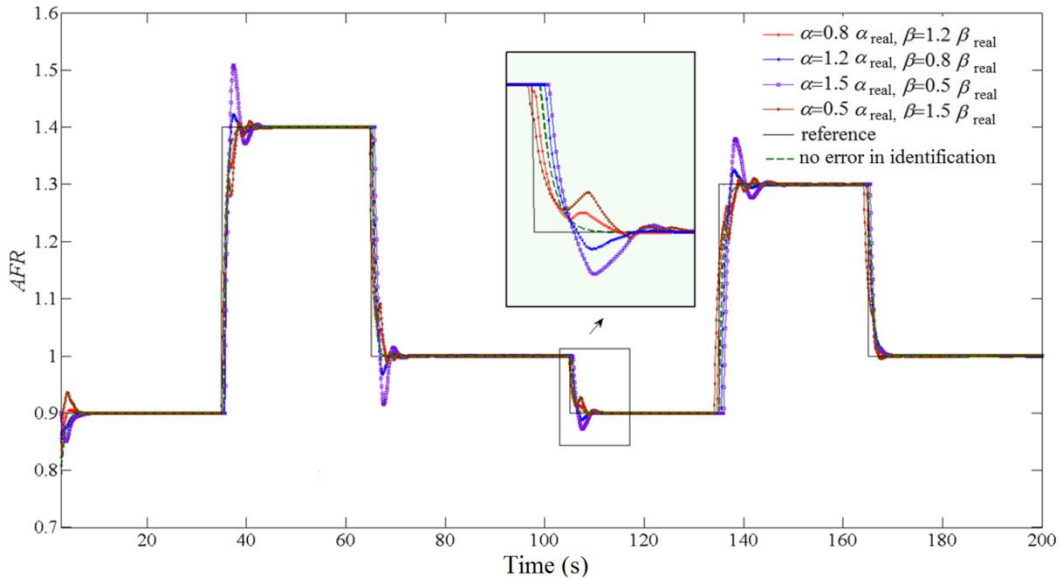


Figure 10. Performance of the enhanced SPBCS for AFR control in the case of error in identification of α and β .

Table 3. Mean of absolute control error with inaccurate parameters.

Control System	MACE for various errors in identification of α and β			
	+30%, -30%	-30%, +30%	-30%, -30%	+30%, +30%
Feedback IMC PID	0.0147	0.0245	Failure	Failure
Conventional IMC SPBCS	Failure	0.0411	0.0582	0.1454
Enhanced SPBCS	0.0090	0.0076	0.0096	0.0076

4. Conclusion

In this paper, two shortcomings of the structure of Smith predictor based control systems were identified: (i) inherited limits of the classical feedback controllers and (ii) asynchrony between the reference and the predictor output.

On this basis, two design recommendations were suggested to enhance SPBCSs: (i) replacing the feedback controller by a feedback-feedforward structure and (ii) replacing reference signal at the current time by the reference signal at the target time of prediction. Within a corollary, the proposed control system

was proved to produce a high performance with guaranteed stability for a wide class of time-delay processes. The enhanced SPBCS does not face issues of common control techniques for time-delay processes, *i.e.* windup, trade-off between performance and steady-state error, and non-minimum-phase systems (the latter is a result of Pade approximation). The proposed control system was assessed for processes with fixed/time-varying delays, in the presence of parameter identification errors and/or severe noises and/or actuator limits. The proposed control system exhibited superior performance in all cases and evidently outperformed a number of well-designed controllers based on Pade approximation or Smith predictor.

Conflict of Interest

The authors declare no conflicts of interest.

Funding

This work was supported by NPRP grant from the Qatar National Research Fund (a member of Qatar Foundation), grant number is 7-829-2-308.

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Appendix: Proof of the Corollary

$$\text{Eq. (1) results in } T\dot{y}(t) + y(t) = bu(t) \quad (\text{A1})$$

Let us consider $\Delta u(t) = u^*(t) - u(t)$.

Subtracting (4) from (A.1) results in the error dynamics:

$$T\dot{e}(t) + e(t) = b\Delta u(t). \quad (\text{A2})$$

$$\text{With the control law of (2), } \Delta u(t) = -Ke(t), \quad (\text{A3})$$

$$\text{then } T\dot{e}(t) + e(t) = -bKe(t). \text{ Consequently, } \dot{e}(t) = -\left(\frac{bK+1}{T}\right)e(t) \Rightarrow e(t) = e(t_0) \exp\left[-\left(\frac{bK+1}{T}\right)(t-t_0)\right]$$

As a result, if $\frac{bK+1}{T} > 0$, the error consistently decreases and the closed loop system is asymptotically stable.