

Simulation of a Multi-Carrier System in a Non-Linear Flat Fading Channel

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محاكاة لنظام متعدد الحوامل في قناة لاخطية ذات خبو مستو

سامح مصطفى

الخلاصة: يتضمن البحث دراسة عن نظام الاتصال اللاسلكي متعدد الحوامل م.س و يتركز على الطبقة الفيزيائية في نظام التضمين المعني. ان النظام المتعدد الحوامل جذب الانتباه كتقنية اتصالات واوجد تطبيقات ملفتة. يعرض البحث ويناقش نظام متعدد الحوامل المعتمد على تحويل جيب التمام المتقطع د.س.ت. تم الإشارة ايضا الى التشابهات والاختلافات مع النظام المعتمد على تحويل الفوريير. ولقد وجد ان انخفاضاً يصل الى ب.د.هـ في القدرة العظمى يمكن كسبه مقارنة مع النظام المعتاد، وان تحليل الاداء فوق قناة ذو خبو مسطح يشير الى ان اداء اشارة متعدد الحوامل ذو التضمين ك.أ.م. في المخطط المعتمد على د.س.ت. افضل مما هو في المخطط المعتمد على ف.ف.ت. بالرغم من اعتماد اداء الاشارة الكبير على بارامترات القناة.

المفردات المفتاحية: إرسال متعدد الحوامل، م س م، متعدد النغمة المنفصل د م ت، متعدد الارسل بتقسيم التردد المتعامد او اف ا دي ام، الذروة الى متوسط القدرة بي اي بي ار في نظام م س، قناة ذو خبو مستو.

Abstract: This paper is on multi-carrier wireless communication system, focusing on the physical layer in such modulation system. Multi-Carrier MC system has received much attention in modern communication technology and is finding attractive applications. The paper introduces and discusses the discrete cosine transform DCT-based MC. Similarities and differences with respect to the Fourier transform-based system are pointed out. It was found that a reduction up to 5dB in peak power can be gained over the conventional system. Also, the simulation over flat fading channel depicted that QAM signaling in MC scheme based on DCT performed better than that in FFT-based scheme, although it depends strongly on the channel parameters.

Keywords: Multi-carrier transmission, MCM, Discrete multi-tone DMT, Orthogonal frequency division multiplexing OFDM, Peak-to average power ratio PAPR in MC system, Flat fading channel Flat fading channel

1. Introduction

The Multi-Carrier MC technology has received much attention in modern communication systems. In MC systems a number of data symbols are transmitted at separated sub-carriers in parallel thus increasing the symbol length, which reduces the sensitivity to interference. Multi-Carrier Modulation MCM divides the available frequency band into a large number of orthogonal tones which can be implemented in all digital realization by exploiting Discrete Fourier Transform DFT methods as in DMT and OFDM multi-carrier modulation Hanzo et al. 2004; Reimers 2001 and Al-Dhahir and Minn, 2006). Discrete Multi-tone DMT a currently standard for Asynchronous Digital Subscriber Line ADSL units manages bandwidth for data transmission. It is one of the technologies that provides high speed Internet access in residence and offices economically via wire technology (Gagnaire, 2003 and Daly 2003). Orthogonal Frequency Division Multiplexing OFDM gives high resistance to fre-

quency selective fading. It is often used in mobile radio systems such as digital video broadcasting DVB/ digital audio broadcasting DAB (Reimers, 2001 and Al-Dhahir and Minn, 2006). It is proposed for 4th generation wireless communication systems (Bria, 2001). However, due to the large number of sub carriers, the MC signal owns a large Peak to Average Power Ratio PAPR. Thus, the transmitted signal is sensitive to non-linear distortions that will degrade the error performance and introduce high adjacent channel interference with spectrum regrowth. Several techniques for reducing PAPR of multi-carriers signal have appeared in the literature (Mustafa, 2007 and Friese, 1996). However, most of these techniques introduce additional complexity. Moreover, there have been intense research efforts aimed at designing different multi-carrier transceiver scheme with different spectral containment. Among those designs are the Discrete Cosine Transform DCT-MCM (Al-Dhahir and Minn, 2006 and Ak-Dhahir and Minn, 2005) and Discrete Wavelet Multi-Tone DWMT (Daly, 2003 and Farhang-Boroujeny and Chin, 2000).

In this paper DCT-based multi-carrier system was con-

sidered. Many simulations have been carried to outline the differences over linear AWGN, non-linear AWGN and a flat faded channel with respect to the DFT-based system. Section II reviews in brief the properties and representations of the DCT, and section III depicts DCT-based scheme. Simulation results and some further discussions of the error performance over different channel parameters are presented in section IV. Finally, section V concludes the results.

2. Discrete Cosine Transform

The DCT has been considered as one of the best tools in digital signal processing and therefore, it has many applications, e.g., in the area of multimedia and telecommunications. In this section, an introduction to the DCT is given in order to provide background and motivation for our work.

The orthogonal DCT is classified into many different types with slightly different even/odd boundary conditions at the two ends of the matrix. The first definitions of the forward and inverse DCT were given by:

$$y_k = \frac{w_k}{\sqrt{N}} \sum_{n=1}^N x_n \cdot \cos \frac{\pi(2n-1)(k-1)}{2N} \quad k = 1, \dots, N \quad (1a)$$

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=1}^N w_k \cdot y_k \cdot \cos \frac{\pi(2n-1)(k-1)}{2N} \quad n = 1, \dots, N \quad (1b)$$

$$\text{where } w_k = \begin{cases} 1 & k=1 \\ \sqrt{2} & 2 \leq k \leq N \end{cases}$$

The DCT operators map an N-size real sequence into another N-size real sequence. It is a linear Fourier-related transformation similar to the DFT using only real numbers of the DFT. By repeating the samples in a time reversed order and performing a DFT on the length sample set a DCT is obtained. Also, if we represent the N-size sequence as $X=[x_1, \dots, x_N]$ and $Y=[y_1, \dots, y_N]$ and denote the NXN DCT matrix by:

$$C = \left\| \frac{w_k}{\sqrt{N}} \cdot \cos \frac{\pi(2n-1)(k-1)}{2N} \right\| \quad (2)$$

Thus Eq. 1 can be rewritten as $Y=XC$ and $X=YC^{-1}$. DCT is computationally simpler and the DCT matrices are orthogonal, i.e. $CC^T=I$, the inverse transform matrix are obtained with a matrix transpose (where T denotes transposition and I is the identity matrix) (Strang, 1999; Oppenheim, *et al.* 1999 and Rao and Yip, 2003).

3. MC System Model

Figure 1 illustrates the MCM system used for multi-carrier modulation. In DCT-based scheme, the IFFT & FFT blocks are simply replaced by an IDCT & DCT or vice versa, respectively, since IDCT matrix is the transpose of DCT matrix.

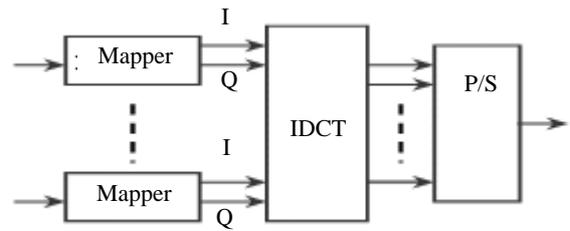


Figure 1. Block diagram of DCT-based MC modulator

In MC-based transceiver, the binary data is encoded into a set of M-PSK or M-QAM symbols, called sub-symbols, and converted into lower rate sequences via serial to parallel conversion, which are then multiplexed by an IDCT. The outputs are serialized and transmitted. At the receiver, inverse operations take place in reverse order. In Fourier-based system, the Hermitian symmetry is enforced to ensure a real waveform at the output of the IFFT. This can be achieved by conjugate mirroring the complex symbols and transmitting zero signals on the DC and Nyquist tones of IFFT block sequence. However, as complex signals are mapped onto orthogonal sub-channels without imposing the conjugate symmetry condition, the multi-carrier system transmits twice the data in twice the bandwidth required when the condition is imposed. Also, it is not necessary in our proposed system, as even a real signal can be mapped onto orthogonal sub-channels.

DCT converts real signals to real signals (as depicted earlier), and hence binary signaling or parsed in-phase and quadrature components of complex data word must be used in each sub-channel individually to keep the same data rate in DCT systems.

4. Simulation Results

Much of this paper compares the error performance of DCT-based MC system, which transmits a real waveform with that of the Fourier-based; therefore the conjugate symmetry condition is imposed on FFT-based system under the same transformation size $N=256$ to provide fairest comparison possible. In the simulation we also considered a system operating without guard space in the resulted time domain symbols.

The scheme was simulated in Matlab ver. 7 with a constant bit allocation of $\log_2(M)$ bits per sub-channel is assumed. The number of simulated bits was about 10^6 bits and the results was computed and averaged over 3 iterations for the demodulator.

4.1 Linear AWGN Channel

The error performance curves presented in Fig. 2 are based on the signal-to-noise ratio SNR of the channel, assuming the channel is a pure AWGN channel. As seen in the figure, a differential phase shift keying (DPSK) in both MC system will limit the number of bits per symbol and results in an 11dB loss in SNR. By comparison the error curves, a gain in SNR can be observed at the low

error rates using DCT-based scheme with QAM modulated tones. The resulted real and imaginary parts from the constellation are used separately to modulate different tones to guarantee real time domain samples without transmitting zero signals on the DC and Nyquist tones as have been applied in IFFT block sequence. Thus the transmission data rate is improved only by $2 \cdot \log_2 M$ than that of FFT-based system.

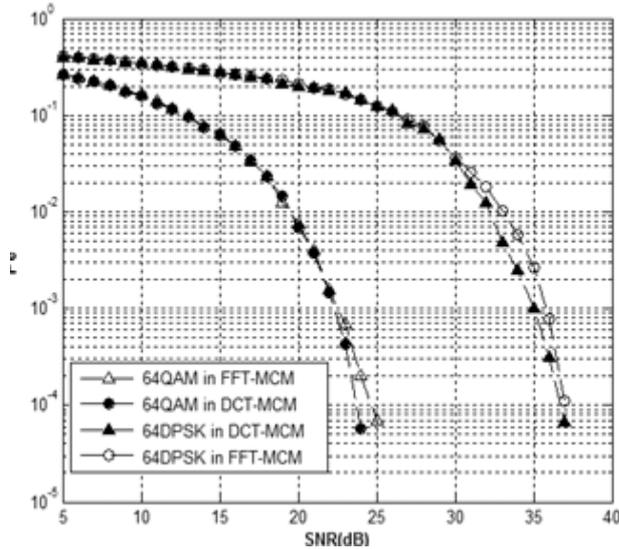


Figure 2. P_e in DCT-based and FFT-based MC scheme versus SNR of linear AWGN channel

However, a DCT scheme can be used with all the sub-carriers allocated to transmit only the estimated phase of the different constellation points (QPSK, DPSK), where the amplitude is constant, thereby improving the data rate to the double.

4.2 Non-Linear AWGN Channel

We also analyzed the distribution of time domain transformed amplitude as depicted in Fig. 3 & Fig. 4. The probability density function (PDF) depends on the probability of occurrence of each discrete sample level.

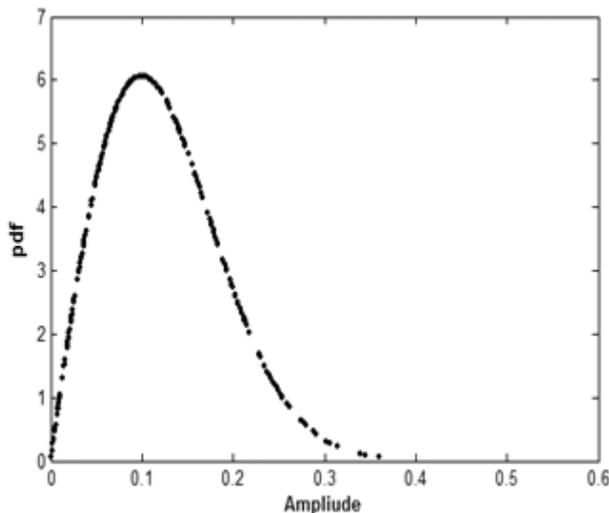


Figure 3. Distribution of DCT-based scheme signal amplitude

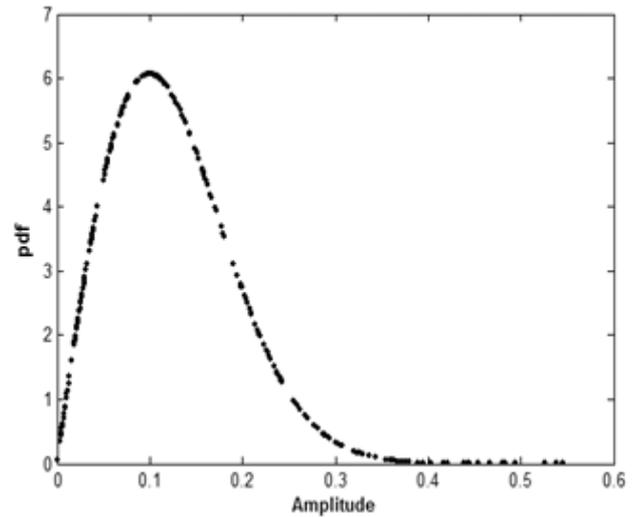


Figure 4. Distribution of FFT-based scheme signal amplitude

The distribution of MC signal with 256 sub-carriers and 16-QAM is founded where the amplitude S_n has Rayleigh distribution with PDF given by:

$$P(s_n) = \begin{cases} \frac{S_n}{\sigma_s^2} e^{-S_n^2 / (2\sigma_s^2)} & S_n \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

where S_n the transformed (using IFFT or IDCT) time domain samples have real parts only as stated before.

For Rayleigh distribution, as seen the signal levels around the mean value have higher probability than other levels, while the occurrence of the large signals has the smallest probability and moreover different level in DCT and FFT-based schemes, as shown in Fig. 3 & Fig. 4. It is reasonable cast a way to change the statistic of the amplitude for the benefit of PAPR reduction. Notice that PAPR is a random variable for each transmitted block. It has been found from a large number of independent runs for the DCT and FFT-based MC scheme that within each block, the peak power is reduced using DCT-based system of about 5dB and a reduction of PAPR of about 1.3dB are achieved.

For the simulated error in Fig. 5 a non-linear power amplifier is applied with AM/AM response of clipping scheme at the saturation point at three different Output Back Off OBO; 4, 5 & 6dB with no compensation of the non-linearity in the conventional MC system. OBO is defined as the ratio of the maximum possible amplifier output power to the average output power. This indicates the power efficiency of the amplifier (Al-Dhahir and Minn, 2006). OBO=6dB guarantees linear transmission while OBO=4 & 5dB in FFT-MCM scheme generates an almost flattened error curves indicating saturation of the power amplifier. These results can be directly compared with the results for MC scheme using DCT. Error curves with OBO=4, 5 & 6dB are also measured. It is found that when OBO=5dB the simulated approaches the linear

transmission due to the overall net improvement in the PAPR using DCT, while when OBO decreases to 4dB the performance diverges from the linear case also, indicating non-linear distortion occurs.

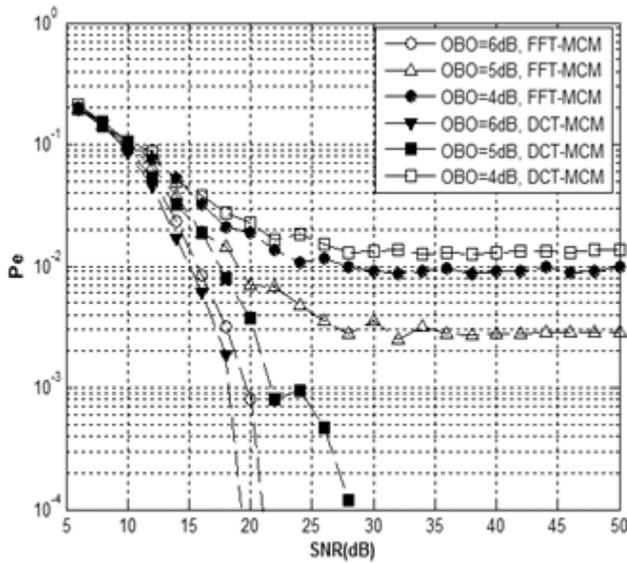


Figure 5. P_e in FFT-based MC scheme versus SNR of non-linear AWGN channel

4.3 Flat Fading Channel

Flat fading has an impulse response given by:

$$g(t, \tau) = \beta(t) \cdot e^{j\theta(t)} \cdot \delta(t) \quad (4)$$

where $g(t, \tau)$ is the impulse response at observation time t to an impulse applied at time $t - \tau$. This channel is considered to be slowly time-varying such that the amplitude $\beta(t)$ and the phase shift $\theta(t)$ can be considered constant during one symbol interval.

To compare and contrast the performance of the schemes over fading channel, the error curves are examined at similar parameters. Fig. 6 shows that, for QAM multi-carrier signal the performance are incompatible exactly for 16-QAM signal where the DCT-based system approaches the linear transmission. The distortion arose from fading channel can be compensated by increasing SNR, while multi-amplitude signaling based on FFT without channel estimation induces irreducible error curves. As depicted the DPSK signals in both MC systems have the same performance depending on the phase tracking of the decoder. The simulated error rates as shown in Fig. 7 are depicted for various flat fading parameters, which affects on the overall DCT-based MC system performance and the error performance limit. It is clearly observed that the amplitude and phase shift of the reflected signals have a much stronger influence on the systems using QAM than on DPSK modulation systems.

The results could be greatly improved to meet the specification requirement that could be implemented in the future to further enhance the simulator, where the results can be extended to any system which uses the MCM technique.

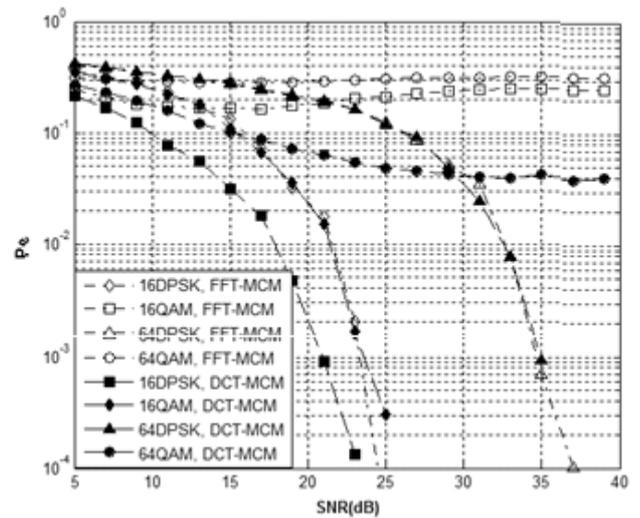


Figure 6. P_e in DCT-based and FFT-based MC scheme versus SNR over flat fading channel

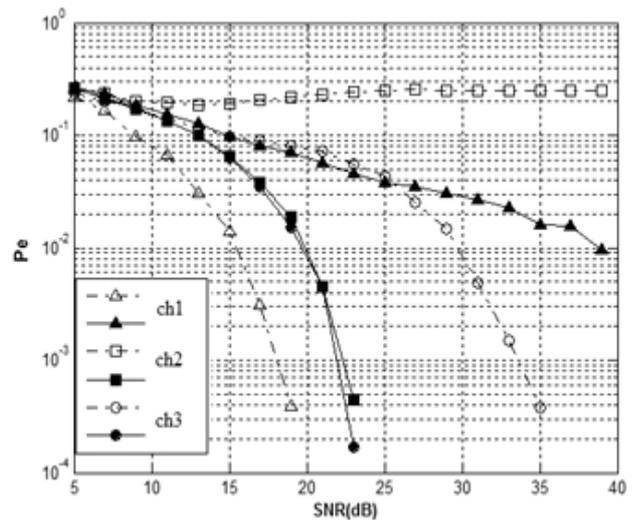


Figure 7. P_e in DCT-based MC scheme versus SNR with 16-QAM (discrete line) and 64-QAM (solid line) modulated sub-carriers over three different flat fading channel characters (ch.1, ch.2, and ch.3)

5. Conclusions

Extensive computer simulations have been carried out to demonstrate and compare the performance of DCT multi-carrier scheme with that of FFT scheme. The results indicate quite similar performance over linear AWGN channel. Concerning the overall system performance, QAM is often more robust and more spectrally efficient than DPSK modulation. Performance enhancement of the proposed system has been recorded over non-linear AWGN channel due to the reduction in PAPR power of the DCT multi-carrier signal. It is more flexible to present linear transmission with no compensation of the non-linearity in the system. Moreover, the simulated results concludes that the new scheme over flat faded channel without channel estimation performs better corresponding to

that of the traditional scheme, especially with QAM modulated tones depending too much on the reflected path amplitude and phase. Such modulated tones in FFT-based scheme results in an irreducible error even at high SNR.

However, the results could be greatly improved to meet the specification requirement that could be implemented in the future to further enhance the simulator, where the results can be extended to any system which uses the MCM technique.

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