

# Vector Directional Distance Rational Hybrid Filters for Color Image Restoration

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## مصفى المتجهات المنطقي المهجن للمسافة الاتجاهية لمعالجة الصور الملونة

**المستخلص:** يقدم هذا البحث نوع جديد من المصفيات غير الخطية يسمى " مصفى المتجهات المنطقي المهجن للمسافة الاتجاهية " لمعالجة الصور متعددة الأطياف. وقد تم تطبيق هذا النوع من المصفيات لحل مشكلات تصفية الصور الملونة. وتعتمد هذه الأنواع من المصفيات على الدوال المنطقية وتتكون من مرحلتين للتصفية حيث يتم الاستفادة من مزايا مصفى المتجهات للمسافة الاتجاهية ومصفى المتجهات للمسافة الاتجاهية موزون المركز مع مزايا العامل المنطقي. ويكون خرج المصفى المقترح ناتج تطبيق دالة المتجه المنطقي على ثلاث مخرجات اثنان مصفيا متجه المسافة الاتجاهي والثالث لمصفى المتجهات للمسافة الاتجاهية موزون المركز وذلك للاستفادة من مزاياهم المختلفة مثل تخفيف الضوضاء والاحتباس اللوني والاحتفاظ بالحواف والتفاصيل. وقد بينت نتائج التجارب ان هيئة مخرجات المصفى المقترح عبارة عن عدد من المصفيات غير الخطية المعروفة لمعالج الصور متعدد الاطياف مثل مصفى المتجه الوسطى ومصفى المتجهات للمسافة الاتجاهية العام ومصفى المتجهات للمسافة الاتجاهية بالنسبة لكافة المعايير المستخدمة.

**المفردات المفتاحية:** الدوال المنطقية، مصفى المتجه المنطقي، مصفى المتجه الوسطى، مصفى المتجهات للمسافة الاتجاهية.

**Abstract:** A new class of nonlinear filters, called vector-directional distance rational hybrid filters (VDDRHF) for multi-spectral image processing, is introduced and applied to color image-filtering problems. These filters are based on rational functions (RF). The VDDRHF filter is a two-stage filter, which exploits the features of the vector directional distance filter (VDDF), the center weighted vector directional distance filter (CWVDDF) and those of the rational operator. The filter output is a result of vector rational function (VRF) operating on the output of three sub-functions. Two vector directional distance (VDDF) filters and one center weighted vector directional distance filter (CWVDDF) are proposed to be used in the first stage due to their desirable properties, such as, noise attenuation, chromaticity retention, and edges and details preservation. Experimental results show that the new VDDRHF outperforms a number of widely known nonlinear filters for multi-spectral image processing such as the vector median filter (VMF), the generalized vector directional filters (GVDF) and distance directional filters (DDF) with respect to all criteria used.

**Keywords:** Rational functions, Vector rational filters, Vector median filters, Vector directional distance filters

## 1. Introduction

Multidimensional signal processing is of paramount importance in application areas such as biomedicine, computer vision, multimedia, industrial inspection and remote sensing. In all these areas end-users and system developers have to work with multidimensional data sets (Haralick and Shapiro, 1992; Plataniotis and Venetsanopoulos, 2000). Noise filtering is an essential part of any image processing based system, whether the final information is used for human inspection or for an

automatic analysis. A number of sophisticated multichannel filters have been developed to date for image filtering (Pitas and Venetsanopoulos, 1990; Astola and Kuosmanen, 1997). Nonlinear filters applied to images are required to suppress the noise while preserving the integrity of edge and other detail information. To this end, vector processing of multichannel images is more appropriate compared to traditional approaches that use instead component-wise operators Machuca and Phillips, (1983). For instance, the vector median filter (VMF) minimizes the distance in the vector space between the image vectors as an appropriate error criterion Astola *et al.* (1990). It inherently utilizes the correlation of the channels and keeps the

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desirable properties of the scalar median; the zero impulse response, and the preservation of the signal edges. VMFs are derived as maximum likelihood estimators for an exponential distribution. They perform accurately when the noise follows a long-tailed distribution (e.g. exponential or impulsive); moreover any outliers in the image data are easily detected and eliminated by VMF's. A second class of filters, called vector directional filters (VDF), uses the angle between the image vectors as an ordering criterion Trahanias *et al.* (1996). The VDF's are optimal directional estimators and consequently are very effective in preserving the chromaticity of the image vectors. A drawback of VDF lies in the fact that they do not consider the magnitude of the image vectors, separating in this way the processing of vector data into directional processing and magnitude processing. However, the resulting filter structures are complex and the corresponding implementations may be slow since they operate in two steps. A third class of filters uses rational functions in its input/output relation, and hence the name "vector rational filter" (VRF) (Khriji *et al.* 1999; Khriji and Gabbouj, 2001). There are several advantages to the use of this function, similar to a polynomial function, a rational function is a universal approximator (it can approximate any continuous function arbitrarily well); however, it can achieve a desired level of accuracy with a lower complexity, and possesses better extrapolation capabilities. Moreover, it has been demonstrated that a linear adaptive algorithm can be devised for determining the parameters of this structure Leung and Haykin, (1994).

In this paper, a novel nonlinear vector filter class is proposed: the class of vector directional distance-rational hybrid filters (VDDRHF). The VDDRHF is formed by three sub-filters (two vector directional distance filters and one center weighted vector directional distance filter) and one vector rational operation. VDDRHF's are very useful in color (and generally multichannel) image processing, since they inherit the properties of their ancestors. They constitute very accurate estimators in long- and short-tailed noise distributions and, at the same time, preserve the chromaticity of the color image. Moreover, they act in small window and require a low number of operations, resulting in simple and fast filter structures.

This paper is organized as follows. Section 2 briefly reviews rational functions and vector rational function filters. The weighted vector directional distance filters are presented in section 3. In section 4, we define the vector directional distance-rational hybrid filter (VDDRHF) and point out some of its important properties; in addition, the proposed filter structures have been considered. section 5 includes simulation results and discussion of the improvement achieved by the new VDDRHF. In order to incorporate perceptual criteria in the comparison, the error is measured in the uniform  $L^*a^*b^*$  color space, where equal color differences result in equal distances (Pratt, 1991).

Section 6 concludes the paper.

## 2. Rational and Vector Rational Function Filters

A rational function is the ratio of two polynomials. To be used as a filter, it can be expressed as:

$$y = \frac{a_0 + \sum_{j=1}^m a_{1j} x_j + \sum_{j=1}^m \sum_{k=1}^m a_{2jk} x_j x_k + \dots}{b_0 + \sum_{j=1}^m b_{1j} x_j + \sum_{j=1}^m \sum_{k=1}^m b_{2jk} x_j x_k + \dots} \quad (1)$$

where  $x_1, x_2, \dots, x_m$  are the scalar inputs to the filter and  $y$  is the filter output,  $a_0, b_0, a_{ij}$  and  $b_{ij}$  ( $i = 1, \dots, m, j = 1, \dots, m$ ) are filter parameters.

The representation described in Eq. 1 is unique up to common factors in the numerator and denominator polynomials. The rational function (RF) must clearly have a finite order to be useful in solving practical problems. Like polynomial functions, a rational function is a universal approximator Leung and Haykin, (1994). Moreover, it is able to achieve substantially higher accuracy with lower complexity and possesses better extrapolation capabilities than polynomial functions.

Straight forward application of the rational functions to multichannel image processing would be based on processing the image channels separately. This however, fails to utilize the inherent correlation that is usually present in multichannel images. Consequently, vector processing of multichannel images is desirable Machuca and Phillips, (1983). The generalization of the scalar rational filter definition to vector and scalar signals alike is given by the following definition:

**Definition 2.1** Let  $X_1, X_2, \dots, X_n$  be the  $n$  input vectors to the filter, where

$X_i = [x_i^1, x_i^2, \dots, x_i^m]^T$  and  $x_i^k \in \{0, 1, \dots, M\}$ ,  $M$  is an integer. The VRF output is given by

$$\begin{aligned} VRF &= RF[X_1, X_2, \dots, X_n] \\ &= \frac{P(X_1, X_2, \dots, X_n)}{Q(X_1, X_2, \dots, X_n)} \\ &= [rf^1, rf^2, \dots, rf^m]^T \end{aligned} \quad (2)$$

where  $P$  is a vector-valued polynomial and  $Q$  is a scalar polynomial. Both are functions of the input vectors. The  $i^{\text{th}}$  component of the VRF output is written as

$$rf^i = \left[ \frac{P^i(X_1, X_2, \dots, X_n)}{Q(X_1, X_2, \dots, X_n)} \right] \in \{0, 1, \dots, M\} \quad (3)$$

where

$$P^i(X_1, X_2, \dots, X_n) = a_0 + \sum_{k=1}^n a_k x_k^i + \sum_{k=1}^n \sum_{l=2}^n a_{k1k2} x_{k1}^i x_{k2}^i + \dots \quad (4)$$

and

$$Q(X_1, X_2, \dots, X_n) = b_0 + \sum_{j=1}^n \sum_{k=1}^n b_{jk} \|X_j - X_k\|_p \quad (5)$$

$\|\cdot\|_p$  is the  $L_p$ -norm, and the square bracket notation used in Eq. (3) above,  $[\alpha]$  refers to the integer part of  $\alpha$ ,  $\alpha \in \mathfrak{R}^+$ .  $b_0 > 0$ ,  $b_{jk}$  are constant, and  $a_{i1,i2,\dots,ir}$  used in Eq. (4) is a function of the input

$$a_{i1,i2,\dots,ir} = f(X_1, X_2, \dots, X_n) \quad (6)$$

When the vector dimension is 1, the VRF reduces to a special case of the scalar RF.

### 3. Weighted Vector Directional Distance Filters

Let  $X: Z^l \rightarrow Z^m$  represent a multichannel image, where  $l$  is an image dimension and  $m$  characterizes a number of channels. In the case of standard color images, parameters  $l$  and  $m$  are equal to 2 and 3 respectively. Let

$$W = \{X_i \in Z^l; i = 1, 2, \dots, N\}$$

represent a filter window of a finite length  $N$ , where  $X_1, X_2, \dots, X_N$  is a set of noisy samples. Note, that the position of the filter window is determined by the central sample  $X_{(N+1)/2}$ . Let us assume that  $w_1, w_2, \dots, w_N$  represent a set of positive real weights, where each weight  $w_j$ , for  $j = 1, 2, \dots, N$  is associated with the input sample  $X_j$ . Introducing the aggregated weighted angular-magnitude distance associated with input sample  $X_i$  gives

$$d_i = \left[ \sum_{j=1}^N w_j \theta(X_i, X_j) \right]^p \cdot \left[ \sum_{j=1}^N w_j \|X_i - X_j\| \right]^{1-p} \quad (7)$$

$$p \in [0, 1] \quad i = 1, 2, \dots, N$$

where,  $\theta(X_i, X_j)$  represents the angle between two  $m$ -dimensional vectors  $X_i = x_{i1}, x_{i2}, \dots, x_{im}$  and  $X_j = x_{j1}, x_{j2}, \dots, x_{jm}$  and  $0 \leq \theta(X_i, X_j) \leq \pi$ .

$$\theta(X_i, X_j) = \cos^{-1} \left( \frac{X_i \cdot X_j^T}{\|X_i\| \cdot \|X_j\|} \right) \quad (8)$$

The power parameter  $p$  is a design parameter ranged from 0 to 1. It controls the importance of the angle criterion versus the distance criterion in the overall filter process. At the two extremes,  $p=0$  or  $p=1$ , the operator behaves as either magnitude processing or directional processing, respectively. The case of  $p=0.5$  gives equal importance to both criteria.

We have adopted a constant operational value  $p=0.25$  as explained by Karakos and Trahanias (1997). This represents a compromise between the different values implied by the different noise models. Moreover, since the performance measures remain practically unchanged for a range of  $p$  values, which includes the value  $p=0.25$ , this is "safe" value independent of the noise distribution.

If ordered distances give the ordering scheme  $d_{(1)} \leq d_{(2)} \leq \dots \leq d_{(N)}$  and the same ordering is implied to the input vector-valued samples  $X_1(d_1), X_2(d_2), \dots, X_N(d_N)$ , it results in the ordered input set  $X_{(1)} \leq X_{(2)} \leq \dots \leq X_{(N)}$ . The output of the WVDDF is the sample  $X_{(1)} \in \{X_1 \leq X_2 \leq \dots \leq X_N\}$  associated with the minimum weighted angular-magnitude distance  $d_{(1)} \in \{d_1 \leq d_2 \leq \dots \leq d_N\}$ . Thus, the WVDDFs are outputting the sample from the input set, so that the local distortion is minimized and no color artifacts are produced (Lukac, 2002).

If all weight coefficients are set to the same value, then all angular distances will have the same importance and the WVDDF operation will be equivalent to the VDDF. If only the center weight is varied, whereas other weights remain unchanged, the WVDDFs perform the Center Weighted Vector Directional Distance Filtering (CWVDDF).

## 4. Vector Directional Distance-Rational Hybrid Filters (VDDRHF)

### 4.1. Design Procedure

The VMF effectively removes impulsive noise and the vector directional filter operates on the directional domain of color images. The filtering schemes based on directional processing of color images may achieve better performance than VMF based approaches in terms of color chromaticity (direction of color data) preservation. In fact, the combination of the direction and the magnitude process (Vector Directional Distance Filter) is suitable for the human visual system and can give a better-balanced result between noise reduction and chromaticity retention. Moreover, as pointed out in (Khriji, *et al.* 1999; Khriji and Gabbouj, 2001), a vector rational filter performs well for relatively high SNR Gaussian contaminated environments.

When both impulsive and Gaussian noises are present, neither the vector rational filter nor VDDF perform well. Thus, it is necessary to use a hybrid structure filter. This structure is made of two filtering stages, as shown in Fig.1. They combine in the first stage the L<sub>p</sub>-norm criteria and angular distance criteria to produce three output vectors in which two vector directional distance filter outputs and one center weighted vector directional distance filter output to eliminate impulsive noise, preserve edges and color chromaticity. In the second stage a vector rational operation acts on the above three output vectors to produce the final output vector. The aim of the final stage, in addition to its detail preserving capability, is to remove Gaussian noise and small magnitude impulsive noise. The VDDRHF is defined as follows:

**Definition 4.1** The output vector  $\underline{y}(X_i)$  of the VDDRHF, is the result of a vector rational function taking into account three input sub -functions which form an input functions set  $\{\underline{\Phi}_1, \underline{\Phi}_2, \underline{\Phi}_3\}$  where the “central one” ( $\underline{\Phi}_2$ ) is fixed as a center weighted vector directional distance sub -filter,

$$\underline{y}(X_i) = \underline{\Phi}_2(X_i) + \frac{\sum_{j=1}^3 \beta_j \underline{\Phi}_j(X_i)}{h + k \cdot D[\underline{\Phi}_1(X_i), \underline{\Phi}_3(X_i)]} \quad (9)$$

where,  $D[.]$  is a scalar output function, which plays an important role in rational function as an edge sensing term,  $\beta = [\beta_1, \beta_2, \beta_3]$  characterizes the constant vector coefficient of the input sub -functions. In this approach, we have chosen a very simple prototype filter coefficients which satisfy the unbiased condition:  $\sum_{i=1}^3 \beta_i = 0$ . In our study,  $\beta = [1, -2, 1]^T$  and  $h$  and  $k$  are some positive constants. The parameter  $k$  is used to control the amount of the nonlinear effect.

The sub-filters  $\underline{\Phi}_1$  and  $\underline{\Phi}_3$  are chosen so that an acceptable compromise between noise reduction, edge and chromaticity preservation is achieved. It is easy to observe that this VDDRHF differs from a linear low - pass filter mainly for the scaling, which is introduced on the  $\underline{\Phi}_1$  and  $\underline{\Phi}_3$  terms. Indeed, such terms are divided by a factor proportional to the output of an edge-sensing term characterized by the function  $D[\underline{\Phi}_1, \underline{\Phi}_3]$ . The weight of the vector directional distance-operation output term is accordingly modified, in order to keep the gain constant. The behavior of the proposed VDDRHF structure for different positive values of parameter  $k$  is the following:

1.  $k \approx 0$ , the form of the filter is given as a linear low - pass combination of the three nonlinear sub - functions:

$$\underline{y}(X_i) = c_1 \cdot \underline{\Phi}_1(X_i) + c_2 \cdot \underline{\Phi}_2(X_i) + c_3 \cdot \underline{\Phi}_3(X_i) \quad (10)$$

where, the coefficients  $c_1$ ,  $c_2$  and  $c_3$  are some constants.

2.  $k \rightarrow \infty$ , the output of the filter is identical to the central sub-filter output and the vector rational function has no effect:

$$\underline{y}(X_i) = \underline{\Phi}_2(X_i) \quad (11)$$

3. For intermediate values of  $k$  the  $D[\underline{\Phi}_1, \underline{\Phi}_3]$  term perceives the presence of a detail and accordingly reduces the smoothing effect of the operator.

Therefore, the VDDRHF operates as a linear low-pass filter between three nonlinear sub-operators, the coefficients of which are modulated by the edge-sensitive component. The proposed structures of the VDDRHF are shown in Figure 1.

### 4.2. Edge Sensor

The proposed edge sensor is written as,

$$D[\underline{\Phi}_1, \underline{\Phi}_3] = \left[ \rho(\underline{\Phi}_1, \underline{\Phi}_3)^p \cdot (\|\underline{\Phi}_1 - \underline{\Phi}_3\|)^{-p} \right] \quad (12)$$

Depending on the value of the parameter  $p$ , the edge sensor behaves as follows:

1.  $p=0$ , the edge sensor is based on the magnitude difference between the vectors in the L<sub>2</sub>-norm sense,

$$D[\underline{\Phi}_1, \underline{\Phi}_3] = \|\underline{\Phi}_1 - \underline{\Phi}_3\|_2 \quad (13)$$

where,  $\|\cdot\|_2$  denotes the L<sub>2</sub>-norm.

2.  $p=1$ , the angles between the directions of the color vectors is now used as an edge sensitivity measure. The goal is to sustain the sharpness of the filtered image by

preserving the transitions detected in the color space, where transactions are represented by the angles between the color vectors. At a fixed luminance, small angles between color-vectors denote “color” homogeneous regions; whereas, large angles indicate edges as given below:

$$D[\underline{\Phi}_1, \underline{\Phi}_3] = \theta(\underline{\Phi}_1, \underline{\Phi}_3)^2 \quad (14)$$

For intermediate value of  $p$  ( $0 < p < 1$ ) both criteria (distance and angle) are used, and in turn they contribute to the filtering process.

### 4.3. The Proposed Filter Structures

The vector directional distance-rational hybrid filters (VDDRHF) are promising detail preserving filtering structures since it was shown that every subfilter is able to preserve signal details within their subwindows Karakos and Trahanias, (1997). VDDRHF are grouped into two classes: unidirectional and bidirectional VDDRHF. The structures for a  $3 \times 3$  window unidirectional and a bidirectional VDDRHF are shown in Figs. 1(a) and 1(b), respectively. Only the points indicated in black in each window are used in the corresponding operation.

Unidirectional VDDRHF are designed to preserve

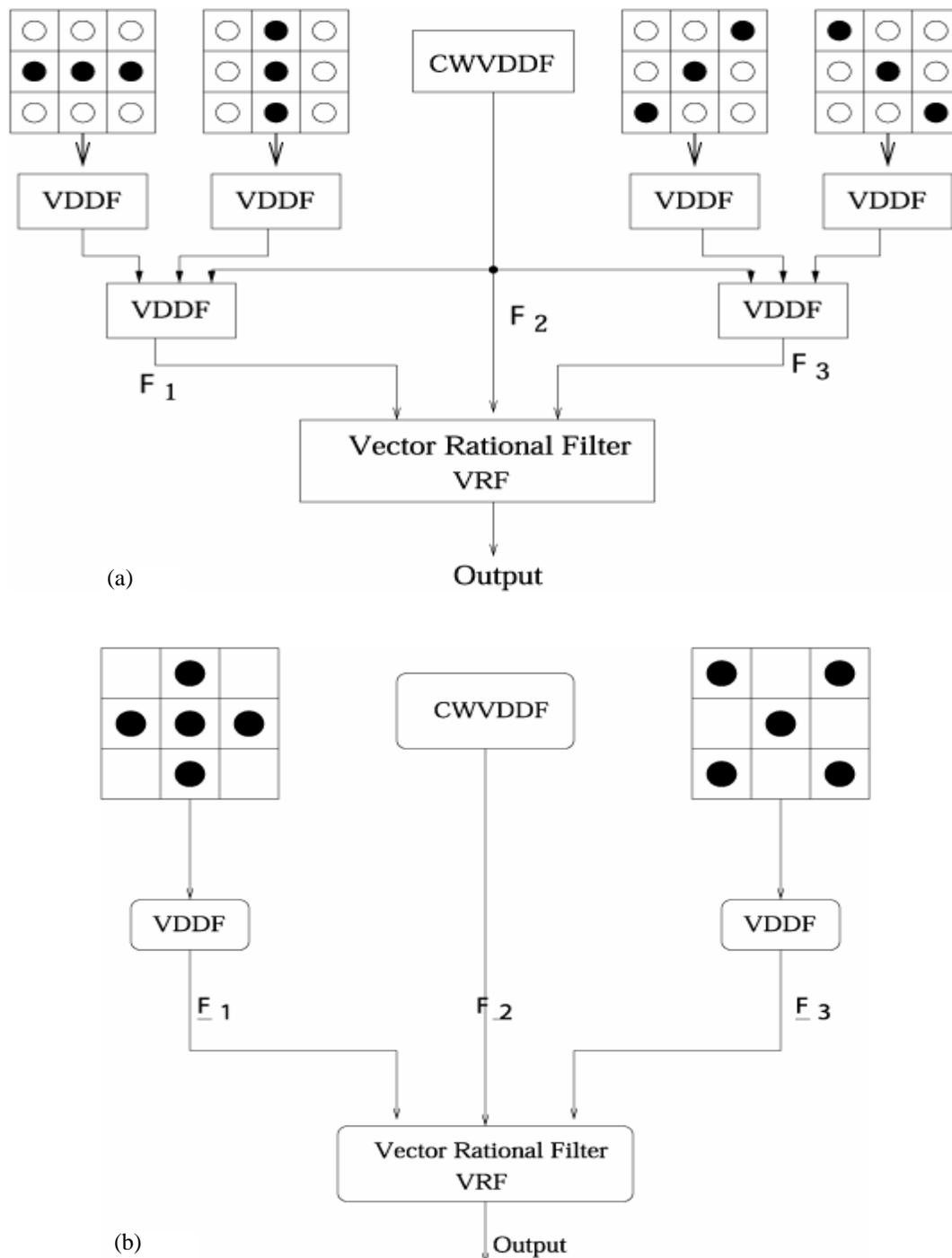


Figure 1. Structures of VDDRHF. (a) The unidirectional structure, (b) the bidirectional structure

image details along the vertical, horizontal and the two diagonal directions. Therefore, the samples of the same value neighborhood must be located along those directions in order to preserve the center sample by unidirectional VDDRHF. Also, bidirectional VDDRHF can preserve details within the two corresponding directions in one operation.

The central subfilter is a center weighted vector directional distance filter characterized by its high detail preservation capability. One of the following three sets of weights can be used depending on the noise properties and the image details Gabbouj *et al.* (1990). Mask M1 emphasizes details in the horizontal and vertical directions, while M2 the two diagonal directions. On the other hand, mask M3 seeks details in all of these directions simultaneously.

$$M_1 : \begin{pmatrix} 0 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad M_2 : \begin{pmatrix} 1 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 1 \end{pmatrix} \quad M_3 : \begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

## 5. Simulation Results

VDDRHF have been evaluated, and their performance has been compared against those of some widely known vector nonlinear filters: the vector median filter (VMF), the distance directional filter (DDF), and the generalized vector directional filter (GVDF) (Karakos and Trahanias, 1997), using RGB color images.

The noise attenuation properties of the different filters are examined by utilizing two color images: (1) part of Pepper image (256x256 pixels); unit (2) the rose image (240x150 pixels). The test images have been contaminated using various noise source models in order to assess the performance of the filters under different scenarios:

- \* Gaussian noise:  $N(0, \sigma^2)$
- \* Impulsive noise: each image channel is corrupted independently using salt and pepper noise. We assume that both salt and pepper are equally likely to occur.
- \* Mixed Gaussian-impulsive noise: the impulsive noise is fixed (salt and pepper 2% in each image channel), while the variance of the Gaussian noise is varied.

The original images, as well as its noisy versions, are represented in the RGB color space. This color coordinate system is considered to be objective, since it is based on the physical measurements of the color attributes. The filters operate on the images in the RGB color space.

A number of different objective measures can be utilized for quantitative comparison of the performance of the different filters. These criteria provide some measure of closeness between two digital images by exploiting the differences in the statistical distributions of the pixel values (Eskicioglu *et al.* 1995). The most widely used meas-

ures are the mean absolute error (MAE), and the mean square error (MSE) defined as:

$$MAE = \frac{1}{MN} \sum_{i=1}^M \sum_{j=1}^N \|y_{i,j} - d_{i,j}\|_1 \quad (15)$$

$$MSE = \frac{1}{MN} \sum_{i=1}^M \sum_{j=1}^N \|y_{i,j} - d_{i,j}\|_2^2 \quad (16)$$

where  $M, N$  are the image dimensions,  $y_{i,j}$  is the vector value of pixel (i, j) of the filtered image,  $d_{i,j}$  is the corresponding pixel in the original noise free image, and  $\|\cdot\|_1, \|\cdot\|_2$  are the  $L_1$ - and  $L_2$ - vector norms, respectively.

Notwithstanding the RGB is the most popular color space used conventionally to store, process, display and analyze color images, the human perception of color cannot be described using the RGB model (Pratt, 1991). Consequently, measures such as the mean square error defined in the RGB color space is not appropriate to quantify the perceptual error between images. It is therefore important to use color spaces, which are closely related to the human perceptual characteristics and suitable for defining appropriate measures of perceptual errors between color vectors. A number of such color spaces are used in areas such as multimedia, video communications (e.g., high definition television), motion picture production, printing industry, and graphic arts. Among these, perceptually uniform color spaces are the most appropriate to define simple yet precise measures of perceptual errors. *The Commission Internationale de l'Eclairage* (CIE) standardized two color spaces,  $L^*u^*v^*$  and  $L^*a^*b^*$ , as perceptually uniform (Gonzales and Woods, 2002).

Conversion from RGB to  $L^*a^*b^*$  color space is explained in detail in (Gonzales and Woods, 2002). RGB values of both the original noise free and the filtered image are converted to corresponding  $L^*a^*b^*$  values for each of the filtering methods under consideration. In the  $L^*a^*b^*$  space, the  $L^*$  component defines the lightness, and the  $a^*$  and  $b^*$  components together define the chromaticity.

In  $L^*a^*b^*$  color space, we computed the normalized color difference (NCD) (Platanotis and Androutsos, 1998) which is estimated according to the following expression:

$$NCD = \frac{\sum_{i=1}^M \sum_{j=1}^N \|\Delta E_{Lab}\|}{\sum_{i=1}^M \sum_{j=1}^N \|E^*_{Lab}\|} \quad (17)$$

where  $\Delta E_{lab}$  is the perceptual color error between two color vectors and defined as the Euclidean distance between them, given by:

$$\Delta E_{Lab} = [(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2]^{0.5} \quad (18)$$

where  $\Delta L^*$ ,  $\Delta a^*$ , and  $\Delta b^*$  are the differences in the  $L^*$ ,  $a^*$ , and  $b^*$  components, respectively.  $E^*_{Lab}$  is the magnitude of the original image pixel vector in the  $L^*a^*b^*$  space and given by:

$$E^*_{Lab} = [(L^*)^2 + (a^*)^2 + (b^*)^2]^{0.5}$$

The results obtained are shown in the form of plots in Figs. 2-4 for the three noise models: Gaussian, impulsive, and

Gaussian mixed with impulsive, respectively. The simulation results of two VDDRHF structures are very close to each other (slight differences), we hence reported only those provided by the bidirectional structure given by Fig. 1(b).

As can be verified from the plots, the VDDRHF filters provide better results than those obtained by any other filter under consideration. Recall that VDDRHF filter uses no information about the type and the degree of noise corruption. Moreover, consistent results have been obtained when using a variety of other color images and the same evaluation procedure.

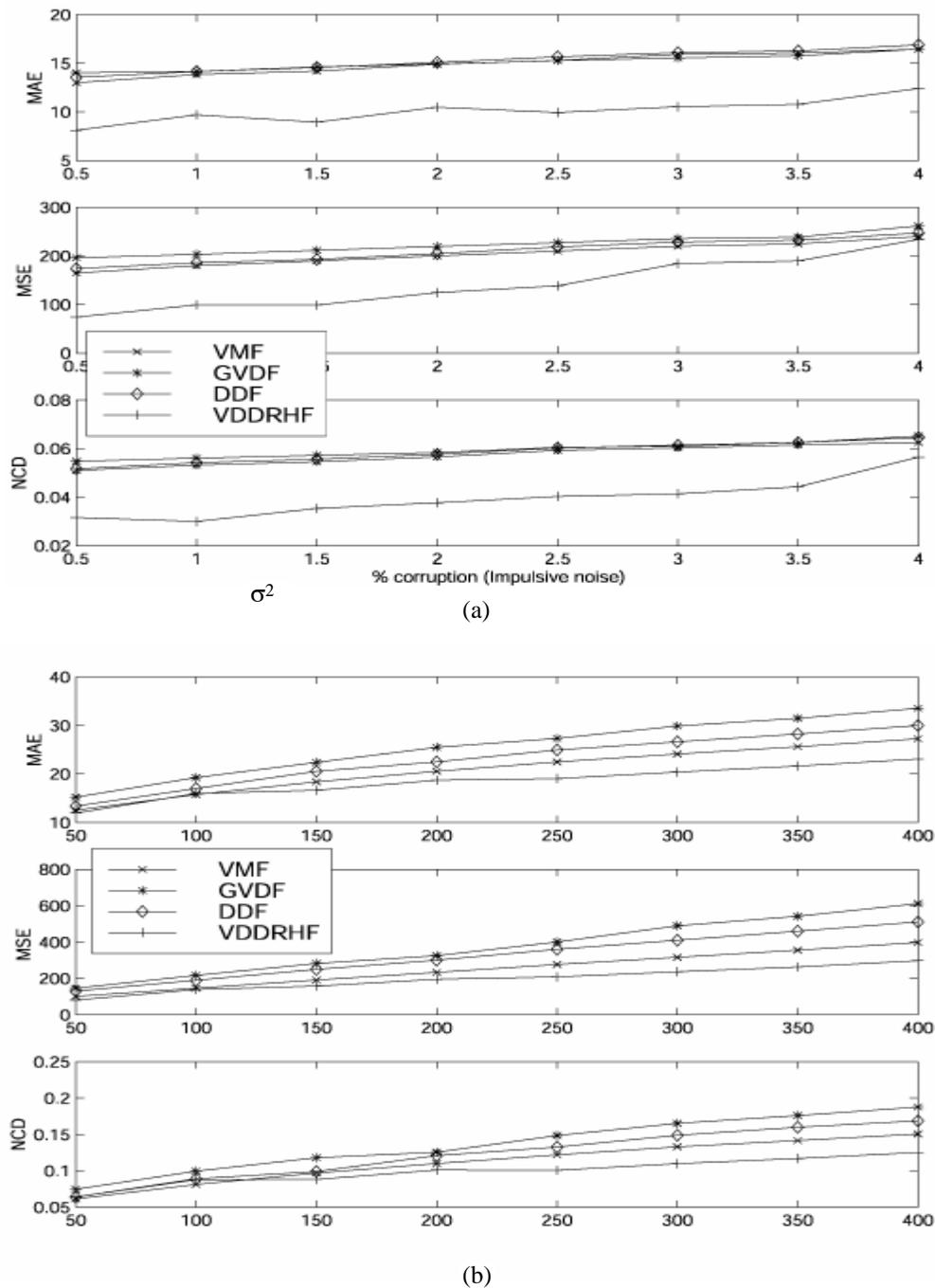


Figure 2. Comparative results for the color test images contaminated by Gaussian noise, (a) Pepper image, (b) Rose image

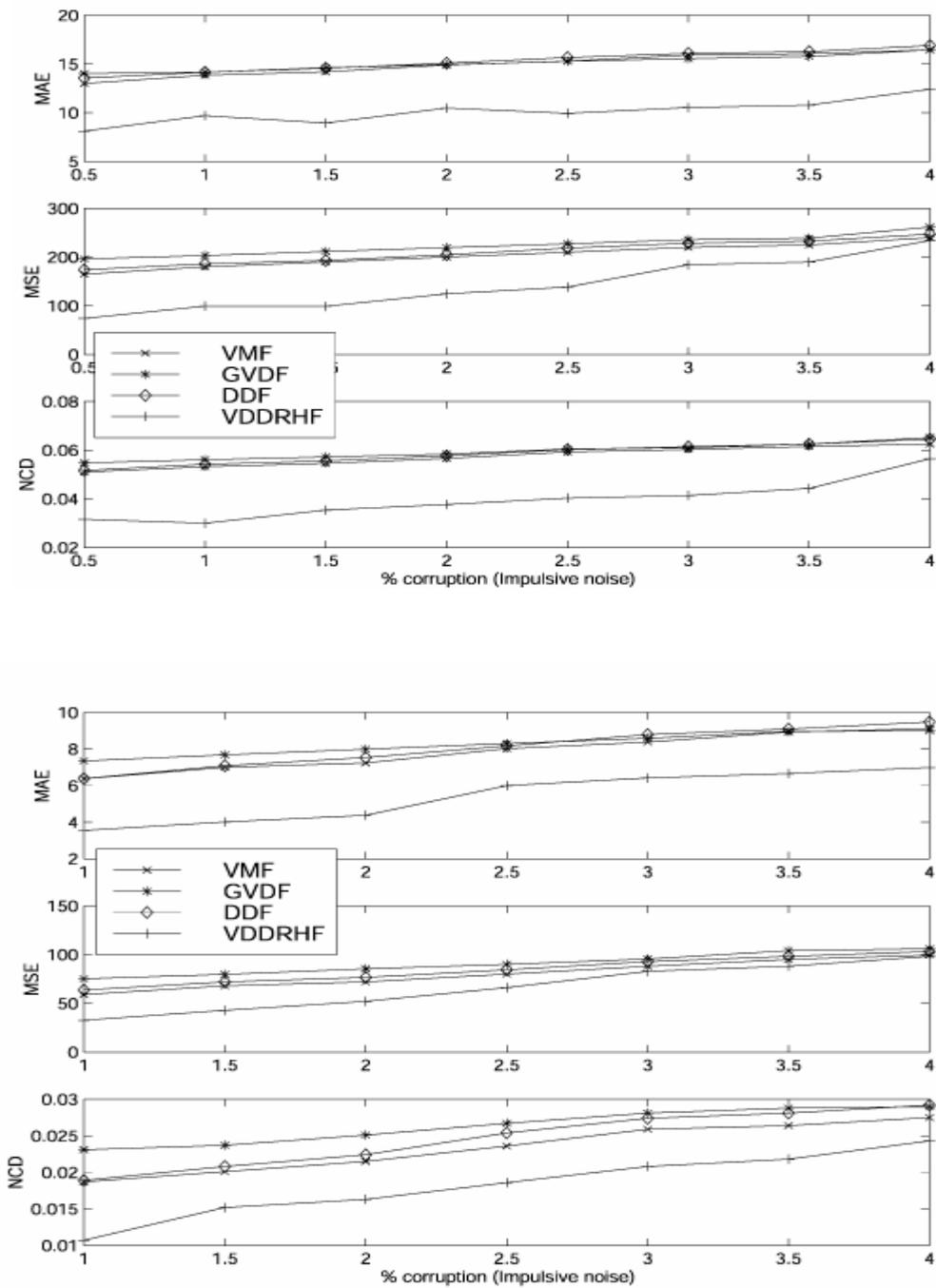
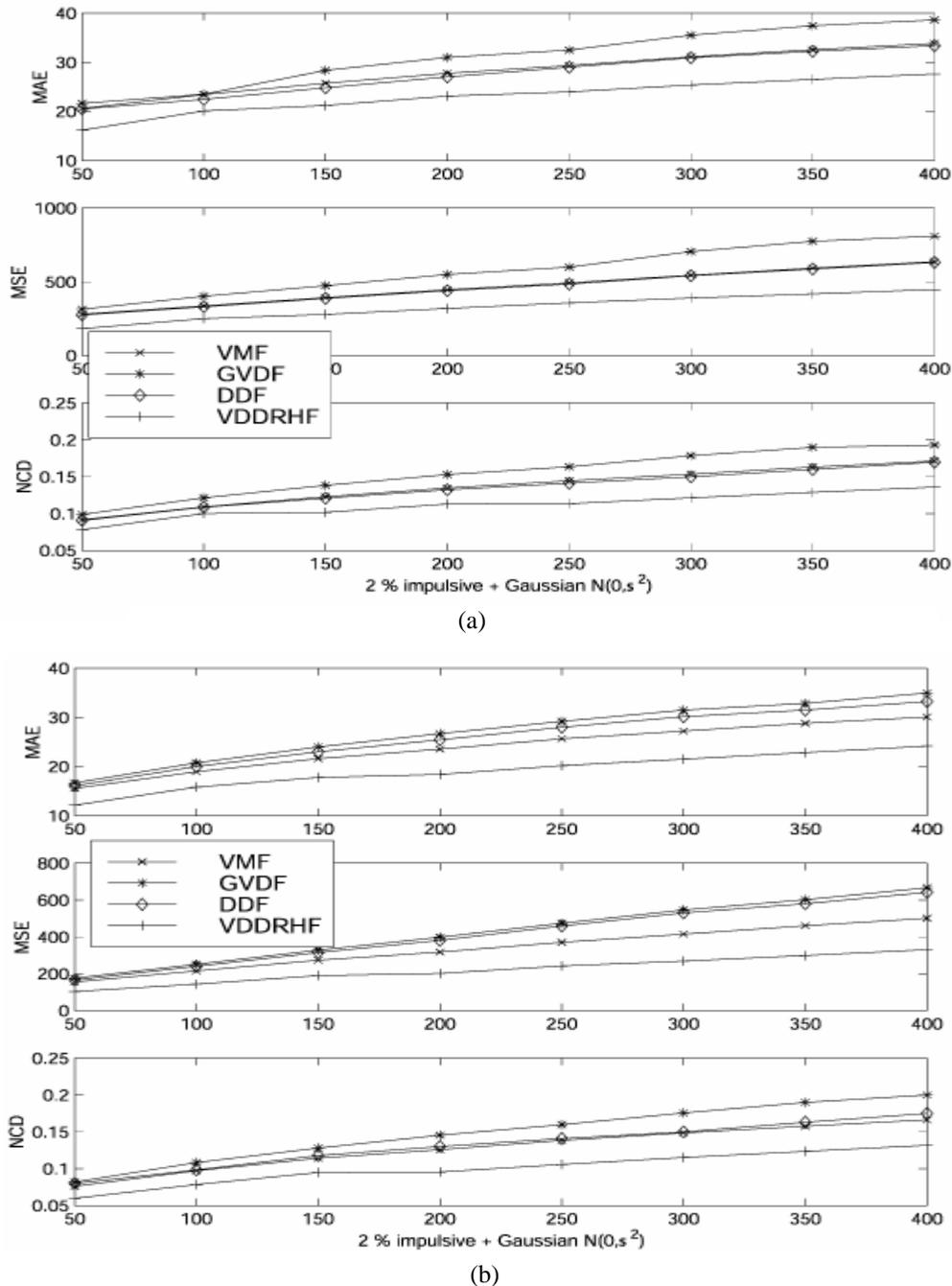


Figure 3. Comparative results for the color test images contaminated by impulsive noise (salt and papper). (a) Pepper Image (b) Rose image)

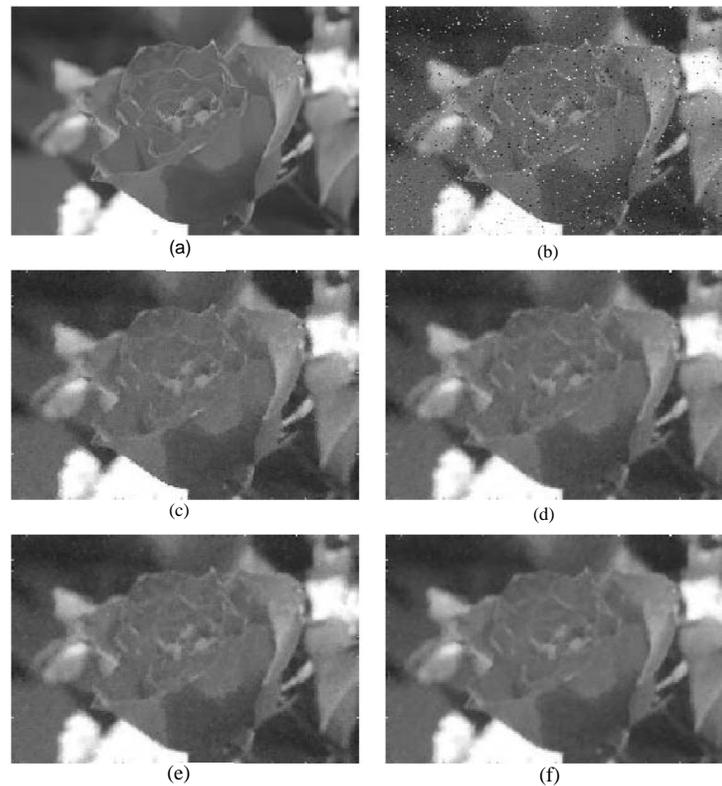


**Figure 4. Comparative results for the color test images contaminated by impulsive mixed noise (salt and pepper 2% in each component, and Gaussian with zero mean and variable variance), (a) Pepper image, (b) Rose image**

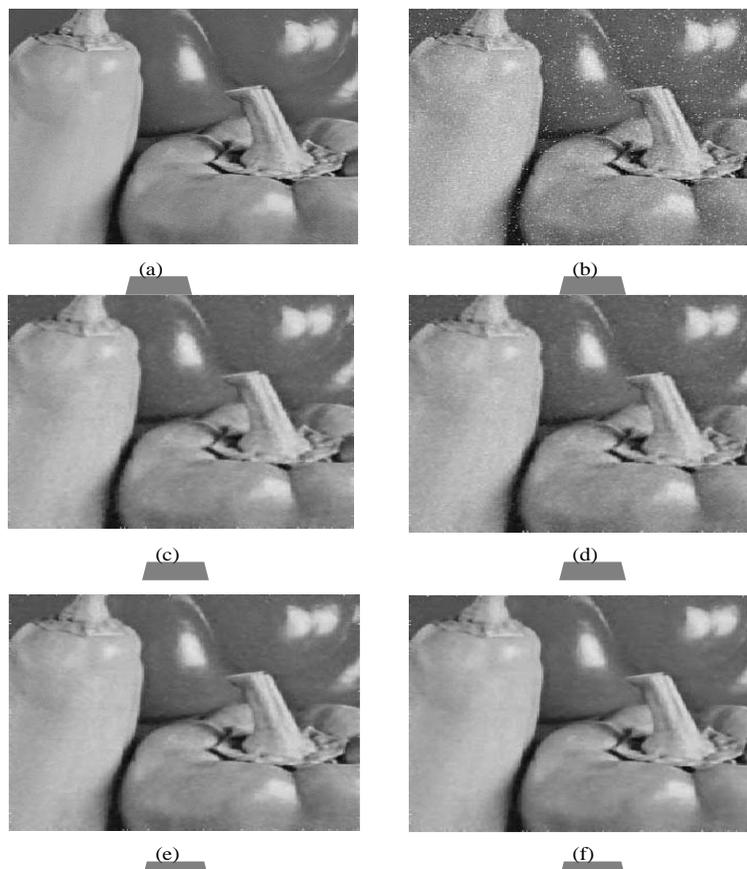
The filtered images are presented for visual assessment, since in many cases they are, ultimately, the best subjective measure of the efficiency of image processing techniques. Figures 5(a) and 5(b) show the noise free and the corrupted rose image by mixed noise (impulsive 2% in each channel and Gaussian  $N(0, 50)$ , respectively. Figures 5(c) and 5(f) represent the filtered images using DDF, GVDF, VMF and VDDRHF, respectively. All the filters considered operate using a 3x3 square processing window.

An additional sample processing results of Pepper image are presented in Figs. 6(a) and 6(f). Figure 6(a)

shows a part (128x128) of original color Pepper image and Fig. 6(b) shows the corrupted version of Fig. 6(a) with additive (4% in each channel) impulsive noise. Figures 6(c) and 6(f) show the results using DDF, GVDF, VMF and VDDRHF, respectively. A comparison of the images clearly favors the proposed VDDRHF over its counterparts (VMF, GVDF, DDF). The proposed VDDRHF can effectively remove impulses, smooth out nominal noise and keep edges, details and color uniformity unchanged as we can see from the related error measures summarized in the plots.



**Figure 5.** (a) and (b) are the original noise free and the contaminated rose image by mixed noise (impulsive 2% in each channel and Gaussian with zero mean and variance 50), respectively. Images in (c)-(f) are the results using DDF, GVDF, and VDDRHF, respectively



**Figure 6.** (a) and (b) are parts of the Original noise free and the contaminated Pepper image with additive (4% in each channel) impulsive noise, respectively. Images (c) - (f) are the results using DDF, GVDF, and VDDRHF, respectively

Furthermore, it is worth mentioning that the proposed filter has comparable or less computational complexity to those used in the comparison, particularly the VDF. The computationally intensive part of the algorithms is the distance calculation part. However, this step is common in all multichannel algorithms considered here. The vector rational operation in the second stage does not introduce significant additional computational cost. In the absence of any fancy or fast algorithms, the number of comparators used in the median filter with a window of size  $n$  is  $N_c = \frac{n(n-1)}{2}$ .

According to Figs. 1(a)-(b), the first stage of the VDDRHF requires 41 comparators: 10 comparators for  $\Phi_1(n_1=5)$ , 10 comparators for  $\Phi_3(n_3=5)$  and 21 comparators for  $\Phi_2(n_2=7)$  with the weights indicated by  $M_1$ ). The second stage requires for the denominator, one multiplication, three additions and one division per output sample. In addition, the different subfilters can be run in parallel reducing the execution time and making the new filters suitable for real-time implementation with digital signal processors. Therefore, our design is simple, does not increase the numerical complexity of the multichannel algorithm and delivers excellent results for complicated multichannel signals, such as color images.

## 6. Conclusions

This paper introduced a new class of nonlinear filters for multichannel image restoration. The VDDRHF filters are two-stage filters. They exhibit very desirable filtering properties and utilize in an effective way the performance of the vector rational function filters and the features of vector directional distance filters. Simulation results and subjective evaluation of the filtered images demonstrated the robustness of the VDDRHF under different noise distributions and have indicated that the VDDRHF outperforms all other filters under consideration. Moreover, the results have shown that VDDRHF has achieved three main objectives: noise attenuation, chromaticity retention and, edge and detail preservation.

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